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Weak antilocalization and spin relaxation in integrable quantum dots

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We study spin relaxation and weak (anti)localization arising from Rashba spin-orbit interaction in ballistic quantum dots with integrable orbital dynamics. We employ a recently developed semiclassical theory for spin-dependent quantum magneto-transport to reveal the dependence of spin dephasing on different types of regular, spatially confined motion. In particular, we analytically derive spin relaxation in circular cavities and compare it with corresponding numerical results. Moreover, we show that different integrable confinement geometries show remarkable differences in their spin evolution.

Key words: *spin relaxation; antilocalization; spin dephasing; circular cavity*

1. Introduction

During the last years research on the effects of spin-orbit (SO) interactions on transport has again received increasing attention. The reason for the revival of such studies, both experimentally and theoretically, lies in an important role SO interactions play in spin electronics and spin-based quantum information processing: in spintronics research, spin interference devices have been suggested, such as spin transistors [1], spin rotators [2], and spin pumps [3], based on SO interactions; in proposals for spin-based quantum computing using quantum dots, SO effects can influence the time scales T_1 for coherent spin relaxation (dephasing) and T_2 for decoherence processes [4].

A useful experimental probe for SO-effects in quantum transport is, on one hand, the beating pattern of Shubnikov-de Haas oscillations in the magneto-conductivity of high-mobility bulk semiconductors [5, 6], which, however, require additional moderate magnetic fields. On the other hand, weak localization (WL) and antilocalization (AL) are prominent examples for SO-interaction-induced quantum interference effects on the conductance in low-dimensional electronic systems [7, 8]. While WL arises

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from the constructive interference of backscattered waves, reducing the conductance for systems with time-reversal symmetry, SO coupling turns constructive interference into destructive interference and hence causes an enhanced conductance, i.e. AL. Recently, weak AL has been reconsidered in a number of corresponding experiments, both for GaAs- [9] and InAs-based [6, 10, 11] two-dimensional (2d) electron gases, as well as for ballistic quantum dots [12, 13]. While SO scattering in (disordered) bulk systems is reasonably well understood [8], the new experiments pose the question of how spatial confinement affects spin relaxation, which is also theoretically of interest and pertinent for the proposals on quantum dot-based quantum computing.

Hence, recently a number of theoretical papers appeared that treat spin relaxation and the interplay between SO and Zeeman coupling in quantum dots [14–17]. Most of these approaches, however, rely on random-matrix assumptions [15, 16, 18, 19], thereby assuming disordered or completely chaotic cavities. In this contribution, we will focus on the opposite case, on integrable confinement geometries giving rise to regular orbital dynamics, where random matrix theory (RMT) is not applicable. To this end, we will employ a recently developed semiclassical approximation to the Landauer formula for spin-dependent quantum transport [17, 20]. This approach constitutes a link between classical orbital dynamics and quantal spin evolution, and it is rather generally applicable to quantum dots with different types of classical dynamics. In a first application of this tool, spin relaxation in extended disordered and confined ballistic systems with Rashba SO interaction has been compared [17]. As a result, spin relaxation is much slower in confined chaotic cavities than for diffusive motion, in accordance with experiments [12] and related theoretical work [14, 18]. Moreover, this preliminary analysis reveals that certain integrable confinement potentials give rise to a saturation of spin relaxation, i.e. an initial spin polarization is preserved to a certain degree even for long times.

Here we will address this interesting finding in detail and present a case study for two representative examples of integrable quantum dots that give rise to rather distinct spin evolution behaviours: a circular billiard with conserved angular momentum and its desymmetrised version, a quarter circle in which angular momentum is no longer a constant of motion. Spin relaxation in the circular geometry has also been recently numerically studied in a closely related semiclassical approach [21]. Here, besides presenting numerical results, we derive an analytical formula for the spin relaxation exhibiting characteristic oscillations. Furthermore, we include the effect of an additional magnetic flux and give predictions for weak (anti)localization for the two geometries.

2. Semiclassical Landauer formula for spin-dependent transport

We consider a Hamiltonian linear in the spin operator $\hat{\mathbf{s}}$

$$\hat{H} = \hat{H}_0(\hat{\mathbf{q}},\hat{\mathbf{p}}) + \hbar\hat{\mathbf{s}}\cdot\hat{\mathbf{C}}(\hat{\mathbf{q}},\hat{\mathbf{p}}) \tag{1}$$

where $\hat{H}_0$ represents the spatial part, including the confinement potential, and $\hat{\mathbf{C}}(\hat{\mathbf{q}},\hat{\mathbf{p}})$ is a rather general vector function of the position and momentum operators $\hat{\mathbf{q}},\hat{\mathbf{p}}$. The term $\hat{\mathbf{C}}(\hat{\mathbf{q}},\hat{\mathbf{p}})$ can include SO coupling, as well as an external (inhomogeneous) magnetic field. For a large number of systems of interest, and usually in experiments, as in those mentioned above

$$\hbar s|\mathbf{C}(\mathbf{q},\mathbf{p})| \ll H_0 \tag{2}$$

even if the spin-precession length, i.e. the distance a particle travels in space during one period of precession of its spin vector, is of the order of the system size. In Eq. (2), s is the particle spin, and the phase-space functions without the hat denote the classical counterparts (Wigner–Weyl symbols) of the respective operators. As a consequence of inequality (2), the back action of the spin dynamics on orbital motion can be neglected. This regime, which we study in this paper, is formally realized by taking $\hbar \to 0$, while keeping all other quantities finite. This corresponds to the orbital subsystem H_0 being in the semiclassical regime, i.e. the typical classical actions $\mathcal{S} \gg \hbar$. In other words, the Fermi wavelength λ_F must be much smaller than the system, which is well fulfilled if the quantum dots are of μm size, as e.g. in Ref. [12].

Hence, only H_0 governs the classical trajectories $\gamma = (\mathbf{q}(t),\mathbf{p}(t))$, along which an effective time-dependent magnetic field $\mathbf{C}_\gamma(t) = \mathbf{C}(\mathbf{q}(t),\mathbf{p}(t))$ acts on spin via the Hamiltonian $\hat{H}_\gamma(t) = \hbar\hat{\mathbf{s}}\cdot\mathbf{C}_\gamma(t)$. We thus employ a *semiquantal* approach, in which the spin dynamics is treated *quantum-mechanically* in terms of a (time-ordered) propagator

$$\hat{K}_\gamma(t) = T\exp\left[-i\int_0^t dt'\,\hat{\mathbf{s}}\cdot\mathbf{C}_\gamma(t')\right] \tag{3}$$

In this way, a weak SO coupling was incorporated into the Gutzwiller trace formula [22] for the semiclassical density of states of s = 1/2-particles [23]. This approach was generalized to arbitrary spin using path integrals in orbital and spin variables [24, 25]; a general semiclassical approach to SO interaction, without relying on inequality (2) and including the back action of the spin onto the orbital degrees of freedom can be found in [24, 26].

Our approach to spin-dependent coherent quantum transport through quantum dots is based on a semiclassical Landauer formula [27, 28] that we generalize to systems with SO and Zeeman interaction. To this end, we start from the Landauer formula in 2d relating the two-terminal conductance $G = (e^2/h)\mathcal{T}$ to the transmission coefficient [29]

$$\mathcal{T} = \sum_{n=1}^{N'}\sum_{m=1}^{N}\sum_{\sigma,\sigma'=-s}^{s}\left|t_{n\sigma',m\sigma}\right|^2 \tag{4}$$

The leads support N and N' open orbital channels m and n, respectively. We distinguish $2s + 1$ spin polarizations in the leads, labelled by $\sigma = -s, ..., s$, and assume that there is no spin-orbit interaction or external magnetic field in the leads. In Eq. (4), $t_{n\sigma',m\sigma}$ is the transition amplitude between the incoming channel $|m,\sigma\rangle$ and outgoing channel $|n,\sigma'\rangle$; a corresponding equation holds for the reflection coefficient $\mathcal{R}$, satisfying the normalization condition $\mathcal{T} + \mathcal{R} = (2s+1)N$, which follows from the unitarity of the scattering matrix.

The transmission and reflection amplitudes can be represented with Green functions. Starting from a path-integral representation of the Green functions, and following the lines of [27] for the spinless case, our semiclassical evaluation yields the spin-dependent transmission amplitudes [20]

$$t_{n\sigma',m\sigma} = \sum_{\gamma(\bar{n},\bar{m})} (\hat{K}_\gamma)_{\sigma'\sigma} \mathcal{A}_\gamma \exp\left(\frac{i}{\hbar}\mathcal{S}_\gamma\right) \tag{5}$$

The transmission amplitudes (and, again, correspondingly the reflection amplitudes) are semiclassically approximated as a coherent sum over classical transmitted (back-reflected) paths at fixed energy. A similar Ansatz has been used in Ref. [30]. The sum runs over classical trajectories $\gamma(\bar{n} = \pm n, \bar{m} = \pm m)$ that enter (exit) the cavity at “quantised” angles Θ_m ($\Theta_{\bar{n}}$), measured from the normal at the lead cross section. For hard-wall boundary conditions in the leads, $\sin\Theta_{\bar{m}} = \bar{m}\pi / kw$ and $\sin\Theta_{\bar{n}} = \bar{n}\pi / kw'$, where k is the wave number, and w, w' are the lead widths. In (5), $\mathcal{A}_\gamma$ is the classical stability amplitude [27], and $\mathcal{S}_\gamma = \int \mathbf{p}\cdot d\mathbf{q}$ is the action along a path γ. For the case of billiards considered below, $\mathcal{S}_\gamma = \hbar k L_\gamma$, where $L_\gamma = vT_\gamma$ is the orbit length, v is the magnitude of the (Fermi) velocity, and T_γ is the time. The entire spin effect is contained in the matrix elements $(\hat{K}_\gamma)_{\sigma'\sigma}$ of the spin propagator $\hat{K}_\gamma \equiv \hat{K}_\gamma(T_\gamma)$ (Eq. (3)) between the initial and final spin states.

Inserting Eq. (5) into (4), we find the transmission $\mathcal{T}$ and reflection $\mathcal{R}$ for spin-dependent magneto-transport in a semiclassical approximation [17]

$$(\mathcal{T}(E,\mathbf{B}),\mathcal{R}(E,\mathbf{B})) = \sum_{nm} \sum_{\gamma(\bar{n},\bar{m})} \sum_{\gamma'(\bar{n},\bar{m})} \mathcal{M}_{\gamma,\gamma'} \mathcal{A}_\gamma \mathcal{A}^*_{\gamma'} e^{(i/\hbar)(\mathcal{S}_\gamma - \mathcal{S}_{\gamma'})} \tag{6}$$

In the case of transmission (reflection), the paths γ, γ' connect different leads (return to the same lead). Equation (6) is still rather general and contains SO and Zeeman interactions. The orbital contribution of each trajectory pair is weighted by the spin *modulation factor*

$$\mathcal{M}_{\gamma,\gamma'} = \mathrm{Tr}(\hat{K}_\gamma \hat{K}^\dagger_{\gamma'}) \tag{7}$$

where the trace is taken in spin space.

3. General discussion

In many physical situations one is interested in the *energy average* of the reflection $\mathcal{R}(E,\mathbf{B})$ (Eq. (6)), subject to an external arbitrarily directed magnetic field **B**. The actions $\mathcal{S}_\gamma(E,\mathbf{B})/\hbar$ are rapidly changing functions of E in the semiclassical limit. Therefore, only the orbit pairs γ,γ' with (nearly) equal actions yield non-vanishing contributions to the energy average.

The *classical* reflection is obtained after summing up the terms $\gamma' = \gamma$ [27], for which the phase in the exponent of Eq. (6) disappears. The modulation factor is then $\mathcal{M}_{\gamma,\gamma} = \mathrm{Tr}(\hat{K}_\gamma \hat{K}_\gamma^\dagger) = 2s+1$, independent of SO interaction, and is reduced to trivial spin degeneracy.

In a system with time-reversal symmetry, i.e. $\mathbf{B} = 0$, each trajectory has a time-reversed partner with identical action. Such pairs contribute to terms with $n = m$ of the semiclassical sum (6) for reflection $\mathcal{R}$ (but not $\mathcal{T}$). The corresponding modulation factor is $\mathcal{M}_{\gamma,\gamma^{-1}}=\mathrm{Tr}(\hat{K}_\gamma^2)$, where γ^{-1} is the time reversal of γ. Upon energy average, the pairs γ,γ^{-1} form a *diagonal* quantum correction $\delta\mathcal{R}_{\mathrm{diag}}$ [27].

The diagonal contribution alone would violate the conservation of current, making it necessary to account for other types of orbit pairs with close actions [27, 28]. No effective way of adding up these terms in a cavity with regular or mixed classical dynamics is known, however. Therefore, we will stay within the diagonal approximation when considering the following examples (see [27] for a similar treatment of the spinless case), and direct the reader to Refs. [17, 28] for a study of chaotic cavities.

In the presence of a constant and uniform magnetic field **B**, the diagonal terms in the sum (6) will be modified. First, due to Zeeman interaction, the modulation factor $\mathcal{M}_{\gamma,\gamma^{-1}}$ is no longer $\mathrm{Tr}(\hat{K}_\gamma^2)$ and should be calculated directly from Eq. (7). Second, the field component B_z perpendicular to the cavity generates an additional Aharonov–Bohm (AB) phase factor $\varphi = \exp\left[(i/\hbar)(\mathcal{S}_\gamma - \mathcal{S}_{\gamma^{-1}})\right] = \exp\left(i4\pi \mathsf{A}_\gamma B_z/\Phi_0\right)$ (the field is assumed to be weak enough to neglect the bending of trajectories by the Lorentz force). Here, $\mathsf{A}_\gamma \equiv \int \mathbf{A}\cdot d\mathbf{l}/B_z$ is the effective enclosed area accumulated along the orbits γ, and $\Phi_0 = hc/e$ is the flux quantum. For a uniform treatment of the SO interaction and magnetic field effects, we thus introduce a generalized modulation factor, $\mathcal{M}_\varphi \equiv \mathcal{M}_{\gamma,\gamma^{-1}}\varphi$. Its average $\overline{\mathcal{M}_\varphi}(L;\mathbf{B})$ over an ensemble of trajectories with fixed length L characterizes the effective evolution of the spinor (both direction and phase) of a particle transported along classical trajectories in a given cavity. The **B**-dependence includes both the AB phase and Zeeman interaction. We estimate the relative quantum correction to reflection as the average $\left\langle \overline{\mathcal{M}_\varphi} \right\rangle_L$ over L

$$\delta\mathcal{R}_{\text{diag}}/\delta\mathcal{R}_{\text{diag}}^{(0)} = \left\langle \overline{\mathcal{M}_\varphi}(\mathbf{B}) \right\rangle_L \equiv \int_0^\infty dL\, P(L)\, \overline{\mathcal{M}_\varphi}(L;\mathbf{B}) \tag{8}$$

where $P(L)$ is the distribution of orbit lengths before they escape from the cavity, and the superscript (0) refers to zero spin and zero magnetic field. Strictly speaking, an expression similar to Eq. (8) has been derived in [27] for a chaotic cavity and used for integrable systems with an appropriate $P(L)$. On an equal footing, we may treat the r.h.s. of (8) as an estimate for the *full* relative quantum corrections to transmission and reflection, $\delta\mathcal{T}/\delta\mathcal{T}^{(0)}$ and $\delta\mathcal{R}/\delta\mathcal{R}^{(0)}$, since the relative diagonal and off-diagonal contributions are the same in the chaotic case [20, 28].

4. Application to integrable billiards

For the remainder of the paper we will focus on the case of spin s = 1/2 and Rashba SO interaction [31], which is often present in 2d semiconductor heterostructures. It is described by an effective magnetic field

$$\mathbf{C} = (2\alpha_R\, m_e/\hbar^2)\,\mathbf{v}\times\hat{\mathbf{z}}, \tag{9}$$

where α_R is the Rashba constant, m_e is the effective mass, $\mathbf{v}$ the (Fermi) velocity, and $\hat{\mathbf{z}}$ is the unit vector perpendicular to the cavity*. In a billiard with fixed kinetic energy, $\mathbf{C}$ is constant by magnitude and its direction changes only at the boundary. We will characterize the SO coupling strength by the mean spin-precession angle per bounce, $\theta_R = 2\pi L_b/L_R$, where L_b is the average distance between two consecutive bounces and $L_R = 2\pi v/C$ is the Rashba length.

Of the two systems chosen for our numerical study, the quarter-circle billiard can be thought of as a „typical“ representative of integrable billiards, while the circular billiard is rather exceptional. In the latter case, owing to angular-momentum conservation, *all* trajectories efficiently accumulate area. Hence, the AB phase grows linearly in time and the spin modulation factor is also affected in a similar fashion (see below).

The dependence of the average modulation factor $\overline{\mathcal{M}}(L) \equiv \overline{\mathcal{M}_\varphi}(L;\mathbf{B}=0)$ on the orbit length for two SO coupling strengths is shown in Figs. 1 and 2. The average was performed over $50,000$ trajectories (in the closed system, i.e. disregarding the leads), with random initial velocity directions and positions at the boundary. Note that as the trace of a unitary matrix, the modulation factor (7) is defined in the interval $[-2, 2]$ (for spin 1/2). All curves in Figs. 1, 2 begin at $\overline{\mathcal{M}}(0) = 2$ due to the initial condition of $K_\gamma(0)$ being a unit matrix. The value $\overline{\mathcal{M}}(L_b) \approx 2 - \theta_R^2$ is also predetermined: before

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* $\alpha_R/\hbar^2$ is kept fixed in the formal semiclassical limit $\hbar \to 0$.

its first encounter with the boundary, a particle moves along a straight line, irrespective of the geometry of the cavity.

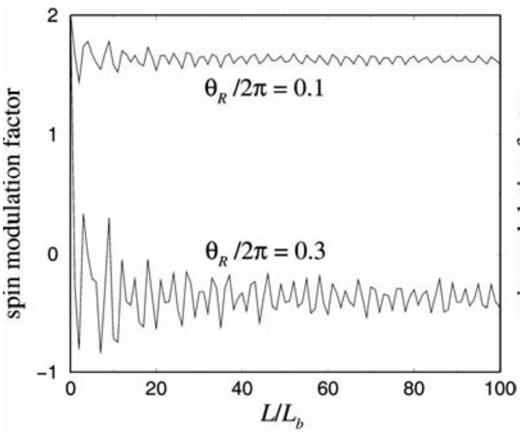

Fig. 1. Average spin modulation factor $\overline{\mathcal{M}}(L)$ for the quarter-circle billiard at two values of spin-orbit coupling strength, $\theta_R/2\pi = 0.1, 0.3$. The orbit length is measured in units of the average distance between two consecutive bounces L_b

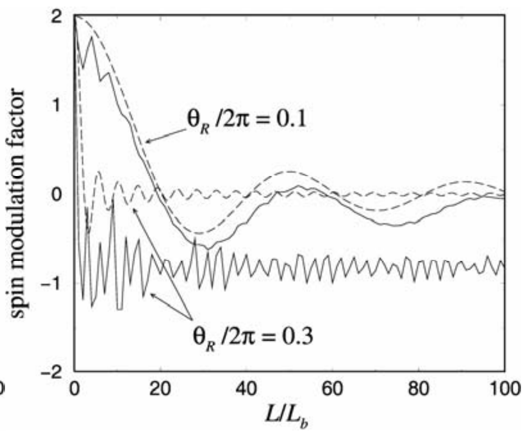

Fig. 2. Solid curves: same as in Fig. 1 for the circular billiard. Dashed curves: analytical prediction by Eq. (10) of the respective values of θ_R

The shape of the billiard becomes important on scales $L >> L_b$. We find that for the quarter-circle billiard (Fig. 1), $\overline{\mathcal{M}}(L)$ oscillates around a constant saturation value. With increasing θ_R, this value decreases down to –1, the level corresponding to a fully randomised spin state [20]. The oscillation frequency is independent of θ_R. The observed behaviour of $\overline{\mathcal{M}}(L)$ can be explained by nearly periodic changes in the spin direction and phase during orbital motion in an integrable billiard. The average over many trajectories results in the saturation value (the remaining oscillations cannot be removed by averaging).

For the circular billiard (Fig. 2), we see a completely different situation. Here, for a weak SO coupling the modulation factor can be estimated as

$$\overline{\mathcal{M}}(L) \simeq 2\frac{\sin x}{x}, \qquad x = \theta_R^2 \frac{Lr}{2L_b^2}, \qquad \theta_R \ll 1 \tag{10}$$

where r is the radius (see Appendix A). The analytical curve (2) (dashed curve in Fig. 2) shows a reasonable agreement with the corresponding numerical results for $\theta_R/2\pi = 0.1$. As θ_R increases, the oscillations of $\overline{\mathcal{M}}(L)$ become irregular and the saturation value falls to approximately –1.

Figure 3 presents the relative quantum correction (8) as a function of θ_R. It is well known that $\delta\mathcal{R}^{(0)} > 0$ [27, 28], which is a consequence of a *weak localization*. The enhancement of reflection due to the quantum interference between different paths in Eq. (6) is responsible for this effect. SO interaction reverses the sign of the quantum correction. This phenomenon is called *weak antilocalization*. According to Eq. (8), AL occurs if the average modulation factor $\overline{\mathcal{M}}(L)$ becomes negative. Contrary to the situation in chaotic quantum dots [17], the shape of the length distribution $P(L)$ is of lesser importance here, since $\overline{\mathcal{M}}(L)$ quickly saturates in most integrable cavities. In our examples, AL occurs in the circular billiard at weaker SO-coupling strengths than in the quarter-circle billiard.

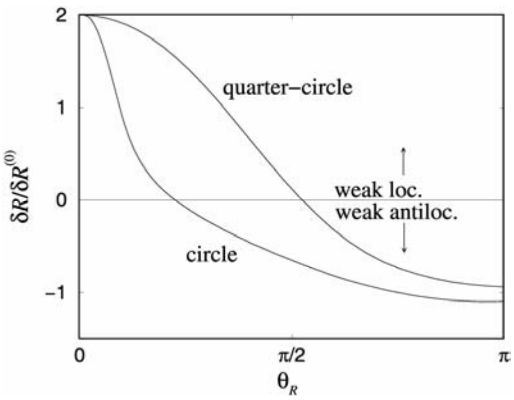

Fig. 3. Relative quantum correction to the reflection $\delta\mathcal{R}_{\rm diag}/\delta\mathcal{R}_{\rm diag}^{(0)}$ (Eq. (8)) vs. spin-orbit coupling strength θ_R for the quarter-circle and circular billiards. Positive (negative) values indicate weak localization (antilocalization). The length distributions $P(L)$ were evaluated at perimeter/lead width equal to 60

A magnetic field AB flux destroys the time-reversal symmetry, thus suppressing the WL and AL. We have also found numerically that Zeeman interaction destroys the AL.

5. Conclusions

We have applied a general semiclassical theory of spin-dependent transport in ballistic quantum dots [17] to 2d cavities with integrable classical dynamics. We compared the properties of two billiards with rather distinct dynamics – the quarter-circle billiard, as a typical integrable example, and the circular billiard, where trajectories accumulate area linearly with time. The two systems have qualitatively different average spin evolutions, which is reflected in the dependences of their average modulation factors on the orbit length. For the circular billiard, we have derived the analytical form of this function for weak spin-orbit coupling. As a consequence of the spin dynamics generated by the orbital motion, AL is predicted to appear in the circular billiard at weaker spin-orbit interactions than in the quarter-circle billiard.

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Appendix A: Average modulation factor in the circular billiard for weak SO coupling

We derive a generalization of Eq. (10) in the presence of an AB flux (neglecting Zeeman interaction). To this end, we apply a unitary transformation [18] $\hat{H} \to \hat{U}^{\dagger}\hat{H}\hat{U}$ to the Hamiltonian (1) with the Rashba interaction, where

$$\hat{U} = \exp\left[i\frac{2\pi}{L_R}\left(q_x \hat{s}_y - q_y \hat{s}_x\right)\right]. \tag{11}$$

Here, q_i are the 2d Cartesian coordinates in the billiard plane, and $\hat{s}_i$ are the respective spin operators ($i = x, y$). Carrying out this transformation to the order θ_R^2, we obtain the new Hamiltonian with a rescaled SO coupling. Its effective magnetic field is

$$\widetilde{C}_z = -\frac{1}{2}\left(\frac{2\pi}{L_R}\right)^2 \mathbf{v}\cdot\left(\hat{\mathbf{z}}\times\mathbf{q}\right), \qquad \widetilde{C}_x = \widetilde{C}_y = 0 \tag{12}$$

With this field, the spin propagator along a trajectory γ becomes

$$\hat{K}_\gamma(t) = \exp\left[i\left(\frac{2\pi}{L_R}\right)^2 \mathsf{A}_\gamma(t)\, \hat{s}_z\right] \tag{13}$$

where $\mathsf{A}_\gamma(t)$ is the area enclosed by the orbit. Clearly, the SO contribution to the semiclassical sum (6) is similar to the AB contribution with the magnetic field $\widetilde{B}_z = \pm\pi\Phi_0/L_R^2$, the sign being dependent on spin polarization [18].

By including the additional external field B_z, we find the generalized modulation factor for a pair of mutually time-reversed orbits of length L to be

$$\mathcal{M}_\varphi(L) = \sum_{\pm} \exp\left[i\frac{4\pi}{\Phi_0}\left(B_z \pm |\widetilde{B}_z|\right)\mathsf{A}_\gamma(L)\right] \tag{14}$$

The enclosed area can be estimated as $\mathsf{A}_\gamma(L) \simeq (M/2m_e v)L$ for an orbit in the circular billiard with angular momentum M. The average of $\mathcal{M}_\varphi(L)$ (Eq. (14)) over M yields the required expression

$$\overline{\mathcal{M}_\varphi}(L;B_z) = \sum_{\pm}\frac{\sin x_\pm}{x_\pm}, \qquad x_\pm = \frac{2\pi}{\Phi_0}(B_z \pm |\widetilde{B}_z|)rL, \tag{15}$$

where r is the billiard radius. Note the partial compensation of the external and SO magnetic fields when one of the $x_\pm$ is zero (cf. [19]).

Here, a final remark is in order. A careful transformation [20] of the modulation factor corresponding to the unitary transformation (11) of the Hamiltonian brings about an additional contribution that is not included in (14). Approximately, it equals $\delta\mathcal{M}_\varphi \simeq -(2\pi/L_R)^2\,|\Delta\mathbf{q}_\gamma|^2$, where $|\Delta\mathbf{q}_\gamma|$ is the distance between the initial and final points of the orbit γ. Neglecting this term is justified, since the relevant orbits start and end within the lead width. In all our numerical calculations of Sec. 4, however, we started and ended the orbits at random points on the boundary. In this case, the correction $\delta\mathcal{M}_\varphi \sim -\theta_R^2$ corresponds to the spin relaxation before the first encounter with the boundary (see above). This contribution is not important for long orbits when $\theta_R \ll 1$. It would be desirable to refine our calculations by checking if θ_R^2-relaxation is present when $\theta_R \backsim 1$ even for closed orbits (as we currently assume).

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