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Methods for estimating the Hurst exponent. The analysis of its value for fracture surface research

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The aim of the work was the selection of the most suitable method for fracture surface analysis using modern tools for mathematical verification of a dataset containing irregular data values and considerable variability of amplitude. The Hurst exponent was used for the verification of the analyzed dataset. The proposed procedure for the Hurst exponent calculations was verified for the profiles obtained from the fracture surface of 18G2A steel and pure iron in the rough state and after wavelet approximations.

Key words: *Hurst exponent; profile; wavelet decomposition*

1. Introduction

The surface obtained during the fracture test is not easy to describe and interpret due to its great level of complexity. Investigation processes usually give a collections of profiles [1]. The Fractal theory application for a description of the nature of the fracture surface has already led to significant developments of the physical interpretation. Application of the wavelet methods could simplify the profile analysis even further [2]. In this work, it has been assumed that the shape of a fracture surface is a natural consequence of the metallic materials crystalline structure, especially for brittle fractures arising from extreme conditions (fast strain rate, low temperature, etc.). The method of light section was used to obtain the shape of profiles from the fracture surface.

When using this method a real shape of the profile is distorted by secondary light beam reflections. In order to remove such reflections, a choice of a suitable filter is necessary. Every profile has to undergo the same procedure of image processing using the same parameters. In the present work, the wavelet decomposition method was applied as the filtration method of the profile.

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Differentiation between chaotic and determined components of the investigated images is another problem in the work presented. Fractals- and multifractals-based theory [3, 4], wavelet decomposition of the profile and estimating of the Hurst exponent have been used for this differentiation [5]. The aim of the study was the selection of the most suitable method for fracture surface analysis using modern tools for the mathematical verification of a dataset containing irregular data values and considerable variability in amplitude. The Hurst exponent analysis is a very popular procedure in many fields of scientific research. In this paper, the efficiency of several of the most popular methods using this exponent have been checked on several selected examples. Three methods were selected for the estimation of the Hurst exponent: the R/S method, the roughness–length (R–L) method and a variogram. The basic dependences have the following forms:

$$\frac{R^*_{(w)}}{S_{(w)}} = \frac{R}{S} = cw^H, \tag{1}$$

$$\overline{S}_{w(i)} = c\left(w_i\right)^{H^*} \tag{2}$$

$$V_{(w)i} = \left\{\left[y(x) - y(x + w_i)\right]^2\right\}^* \tag{3}$$

$$V_{(w)i} = cw_i^{2H} \tag{4}$$

where: $R_{(w)}$ is the difference of the height of the profile in the window, c – the constant, $S_{(w)}$ – the standard deviation, H – the Hurst exponent, w_i – the width of the window (section, lag), x and y are the data coordinates. The exponent H is the slope of an interpolated straight line in the log–log system. (The mark * denotes the mean value for the data on each of the windows).

The R/S method [6, 7] is commonly perceived as the most suitable for the time series analysis on the stock market or an optimal volume of water reservoirs, because it presents the relationship between irregular (singular) rescaled ranges, signal value and their local statistical properties relative to the scale factor. There is ample evidence of the popularity of this method.

The roughness–length method [8, 9] is the most popular for describing the roughness of surfaces and is descended from RMS profile analysis or calculations of the R_q – surface roughness parameter. The R–L method permits the description of profiles as the relationships between local statistical properties (standard deviation) versus the scale factor (length of part of the profile).

Variogram is similar to the roughness–length method because it uses Variance for the description of the average trend versus the length of windows (lags). The variogram method is the most popular in earth sciences or geomorphology and is very useful for surface approximations for sparse datasets [10].

2. Results

Available libraries of signal processing algorithms make it possible to choose a signal (profile) trace with established the Hurst exponent values. Testing procedures typically use values of $H = 0.3$, $H = 0.5$ and $H = 0.7$. The tested signal (profile) samples were derived from Benoit 1.3 [11] and the tested profiles from fracture surfaces were obtained from a 18G2A steel sample (impact test, −70 °C) and pure Iron 99.65 Fe (impact test, −191 °C). Figure 1 presents the results of the Hurst exponent calculations for the tested signals and the profile from the pure iron sample.

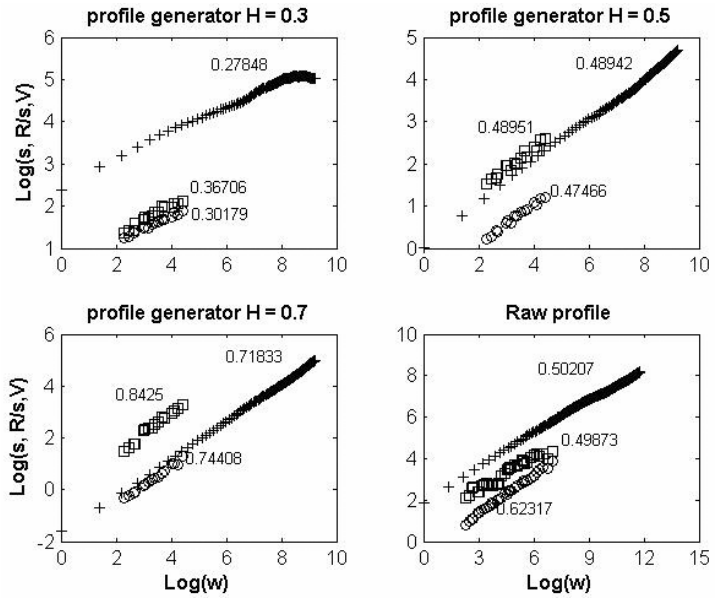

Fig. 1. Results of Hurst exponent calculation for investigated traces. Numbers beside the curve are linear estimation of slope. Marks denote: ○ – RL method, □ – R/S method, + variogram

It can be seen that the same tested trace (profile) gives different results of the Hurst exponent. The average errors of the calculations – the difference between the generated and the computed value of the Hurst exponent were analyzed. The error reached a maximum for the R/S method, the roughness–length and variogram gave very similar and low error values. For several of the calculated raw profiles from both the iron and steel surfaces the results vary. Exponents that are similar can be detected with the R/S method and variogram rather than the roughness–length method. Firstly, the number of points for testing traces ($H = 0.3$, $H = 0.5$ and $H = 0.7$) were 1000, for raw profiles ordinarily more than 3650 points.

Secondly, for the variogram usually the calculated maximum lag values are about 10% of the number of profile points, and for computer generated profiles the maximum lag = 100, and for raw profiles the maximum lag = 366. The other problem was

extraction of optical profiles from the fracture surfaces of the iron and steel samples. Preparation procedure between the raw profile and their graphical representation is shown in Fig. 2 [12].

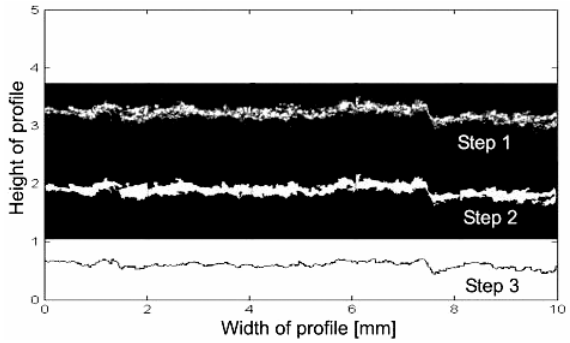

Fig. 2. Three steps profile preparations. Step 1 – acquisition of profile from dual lenses microscopy (originally for one profile we need 5 separated pictures). Step 2 – picture preparation on PhotoShop and Fovea Pro 3.0 (threshold and fill holes procedure). Step 3 – Profile computation and mathematical stitching on Matlab 7

The aim of the research was to find the maximum level of decomposition (with minimum changes of the Hurst exponent values. In this work Symlets3 (Sym3) and Debouchies4 (db4) wavelets were checked. Results of the investigation for R–L method are presented in Fig. 3. All the calculations were made using Matlab 7, where the algorithms were implemented [1–4].

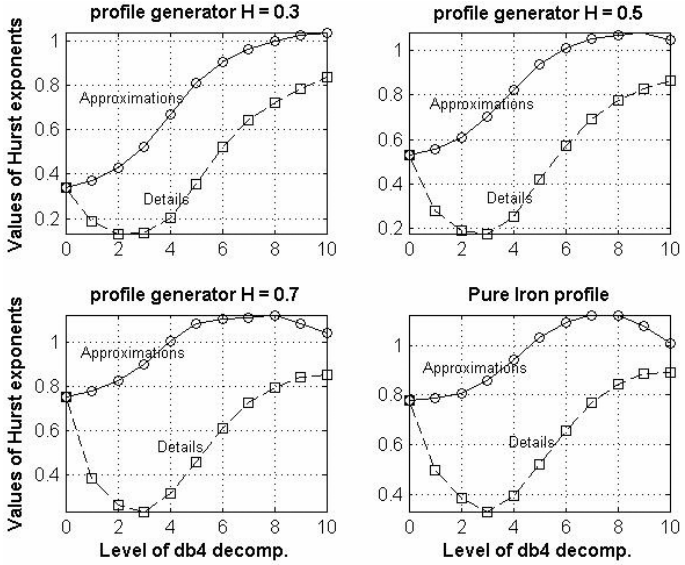

Fig. 3. Values of the Hurst exponent for R–L method, calculated with approximations and details after wavelets decomposition traces and profiles in Fig. 1

In Figure 3, each variant of the investigated profiles is presented. Of course, the results of the wavelet decomposition with levels higher than five or six do not make physical sense.

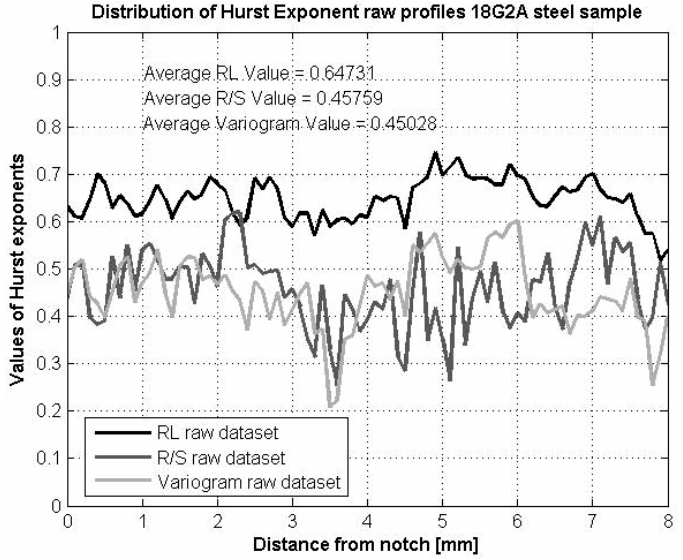

Fig. 4. Distribution of the Hurst exponent calculated on RL, R/S and Variogram algorithms for fracture surface of 18G2A steel sample, broken (17 J energy, – 70 °C). Raw profiles were calculated

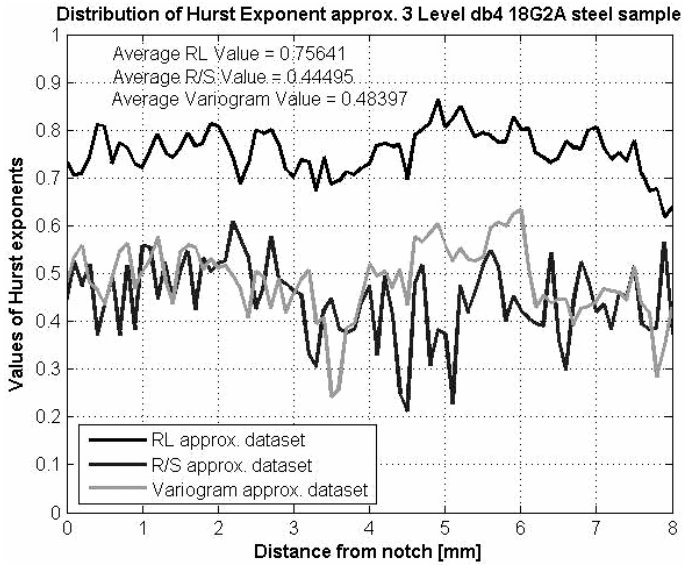

Fig. 5. Distribution of the Hurst exponent calculated on RL, R/S and Variogram algorithms for fracture surface of 18G2A steel sample, broken (17 J energy, 70 °C). Approximations of third level db4 profiles were calculated

Figures 4 and 5 present the distribution of the Hurst exponent calculated for the brittle fracture surface of 18G2A steel sample with a notch at the end of the sample. The distance between profiles was 0.1 mm and one pixel resolution was equal to 0.00268 mm. Figure 4 indicates the raw profile, Figure 5 concerns approximation of the same profile after a third level decomposition by db4 wavelets.

The presented results are controversial. Because the methods of calculation used interpret different physical properties, more sophisticated tools are needed, or stronger theoretical assumptions are required, for correct and precise interpretation. The procedure of scale range adjusting in the R/S method can affect the obtained results. The relationships between decomposition level and the R/S Hurst exponent's for approximations and details are under investigation and will probably explain the shapes of the distribution and dispersion as shown in Fig. 5.

3. Conclusions

The methods give described considerably different results. For two methods (R/S and variogram), the average Hurst exponent oscillate around the value of 0.5. The very convenient and popular roughness–length method gives a slightly higher value. Additionally, it is sensitive to high frequency signal removing processes. Nevertheless, locally, a part of the surface profile reveals significant dispersion of the Hurst exponent (especially the R/S method). In spite of similar theoretical foundations, the practical implementation of these methods is dependent on the physical nature of the investigated occurrence. Probably the R–L method offering the best approximation for the investigated case. The calculation is expensive in terms of time and computational power, but gives a very attractive possibility for analysis of profiles and images. The knowledge base in this scientific discipline is growing fast, especially in the areas of interpretation and algorithms of calculation. These facts create a new field and the possibility for materials science investigations.

References

[1] UNDERWOOD E.E., BANERJI K., Q*uantitative Fractography*, ASM Handbook, Vol. 12, Electronic Edition by ASM International, Metal Park Ohio, 1999.
[2] BIAŁASIEWICZ J., *Falki i aproksymacje*, WNT, Warszawa, 2000.
[3] SIMONSEN I., HANSEN A., NES O.M., Phys. Rev. E, 58 (1998), 2779.
[4] RELJIN S., RELJIN B.D., Arch. of Oncology 10 (2002), 283.
[5] MISTI M., MISTI Y., OPPENHEIM G., POGGI J.M., *Wavelet Toolbox Users Guide*, Version 2, The MathWorks Inc., 2002.
[6] *Estimating the Hurst Exponent*, http://www.bearcave.com/misl/misl_tech/wavelets/hurst/.
[7] HURST H.E., BLACK R., SINAIKA Y.M., *Long-Term Storage in Reservoirs: An experimental Study*, Constable, London, 1965.
[8] *Roughness-Length Method for Estimation Hurst Exponent and Fractal Dimension of Trace*s, Help Benoit 1.3 version Software, TruSoft International Inc., 2003.

[9] GÓRECKA R., POLAŃSKI Z., *Metrologia warstwy wierzchniej*, WNT, Warszawa, 1983.
[10] *Surfer 8 User Guide*, Chapter 5. *Variograms*, Golden Software Inc., 2004.
[11] *Self -Affine Trace Generator*, Help Benoit 1.3 version Software, TruSoft International Inc., 2003.
[12] WAWSZCZAK J., Problemy Eksploatacji, 51 (2003), 153.

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