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# Nonlinear current oscillations in the fractal Josephson junction

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In the present paper, a generator of chaotic oscillations based on the Josephson tunnel junction of the S–NF–S type, where S designates a superconducting material and NF is a thin fractal resistive layer, is offered. The nonlinearity and memory properties of the fractal junction allow us to use such junction as an effective generator of chaotic signals with a controllable statistical structure. Such a generator of chaotic signals can serve as a technological basis for chaotic communications.

Key words: *Josephson junction; fractal; nonlinearity; chaotic communications*

## 1. Introduction

The physics of electronic transport through nanodevices has generated an active interest in recent years [1, 2]. The number of applications of composite materials with complex fractal structures has recently increased in many areas of physics and technology. Fractal materials are characterized by macroscopic properties (heat conductivity, electric conductivity, diffusion, etc.) depending on fractal characteristics such as the fractal dimension of a material and its porosity [3]. It is very important, in particular, that fractal materials possess nonlinear electric conductivity. One of their basic properties is irreversibility, and we see [4] that irreversibility characterizes stochastic processes. These distinctive features of fractal materials allow us to hope that they can be effectively used in many radio-physical devices.

In the present paper, presented is a generator of chaotic oscillations based on the Josephson tunnel junction with a fractal non-conductive layer. Current fluctuations in

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a contour with the Josephson tunnel junction of the S–NF–S type (see Fig. 1), where S designates a superconducting material and NF – a thin fractal resistive layer. The nonlinearity and memory properties of fractal junctions allow us to use such a contact as an effective, easy to modify generator of chaotic signals with a controllable statistical structure. Such a generator can serve as the technological basis for the chaotic communication [5].

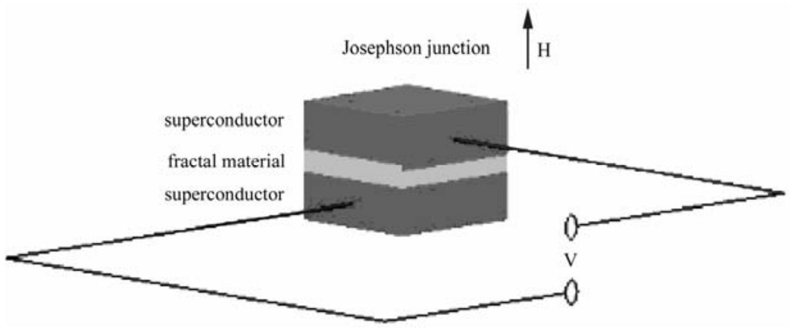


Fig.1. Scheme of the junction

## 2. Results

Let an insulating layer in a tunnel junction consist of a mixture of two components, such as a conducting and a dielectric one. If $\sigma_m$ is the conductivity of the metal component, and $\sigma_D$ the conductivity of the dielectric component, then the effective conductivity $\sigma_e$ of the two-component infinite medium in a threshold behaviour depends only on $\xi = \sigma_D/\sigma_m$ and is described by the power law

$$\sigma_e = \sigma_m \xi^u \tag{1}$$

where $u$ is a critical index. In the two-dimensional case, $u$ is equal to 1/2 [4]. The conductivity at small values of $\sigma$ decreases, and it is shown [3] that nonlinear effects in the conductivity $\sigma$ at the phenomenological level are described by the effective linear conductivity $\sigma_e$ and nonlinear conductivity $\alpha_e$

$$\sigma = \sigma_e + \alpha_e E^2 \tag{2}$$

where $E$ is the average electric field, and the nonlinear conductivity $\alpha_e$ is described by a power dependence on $\xi$:

$$\alpha_e \approx \alpha_D \xi^{-w} \tag{3}$$

where for a thin layer $w$ = 1.5. We should emphasize that the nonlinear conductivity of an isolated fractal layer increases when $\xi \to 0$, and the effective field of nonlinearity decreases, i.e. the fractal layer becomes quasi-nonlinear in a threshold behaviour. This property of the nonlinear conductivity of fractal materials creates additional nonlinearity when they are in contact with two superconductors and allows the properties of this junction to be controlled.

As is known, the system consisting of two weakly connected superconductors with the applied potential difference $U(t)$ is phenomenologically described by the difference in the phases of the wave functions in the superconductors with regard to

$$\frac{\partial \varphi}{\partial t} = \frac{2e}{\hbar} U(t) \tag{4}$$

and by existence of an oscillating superconducting part of the current $I_s$ within the full current of the system $I$ (due to the connection between the superconductors):

$$I_s = I_c \sin\left(\varphi(t)\right) \tag{5}$$

Taking into account that the junction possesses a capacity $C$, we add the equation for currents in the circuit to the obtained relations. It follows that the sum of the superconducting current, the conductivity current $\frac{\hbar}{2e}\frac{\partial \varphi}{\partial t}\frac{1}{R}$ and the displacement current $\frac{\hbar C}{2e}\frac{\partial^2 \varphi}{\partial t^2}$ is the full current in the circuit [4]:

$$\frac{\hbar C}{2e}\frac{\partial^2 \varphi}{\partial t^2} + \frac{\hbar}{2e}\frac{1}{R}\frac{\partial \varphi}{\partial t} + I_c \sin\left(\varphi(t)\right) = I\left(t\right) \tag{6}$$

Taking into account equations (4) and (6) and the conductivity nonlinearity of a fractal isolating layer (3), we obtain a system of first-order equations for the potential difference $U(t)$ on the tunnel junction. This system consists of Equation (4), which connects the phase and potential difference, and an equation for the phase:

$$\frac{\hbar C}{2e}\frac{\partial^2 \varphi}{\partial t^2} + \frac{\hbar}{2e}\frac{1}{R_0}(\xi^{1/2} + \alpha_D \xi^{-3/2} \alpha_e U\left(t\right)^2)\frac{\partial \varphi}{\partial t} + I_c \sin\left(\varphi(t)\right) = I\left(t\right) \tag{7}$$

We can see that these equations represent equations for a nonlinear oscillator of the Van der Pole type, with nonlinear decay and an external force. The external force acting on the oscillator is defined by a current $I$ flowing through the junction, which can be constant or variable. If the current is a harmonic function of frequency close to the resonant frequency of the Josephson junction, then the system (7) can have both regular solutions and chaotic solutions of a complex structure.

The phase trajectory of the solutions of the system, when a normal metal is used instead of a fractal insulating layer in the tunnel junction, is shown in Fig. 2. As is seen, the fluctuations are similar to those for the ordinary Van der Pole oscillators.

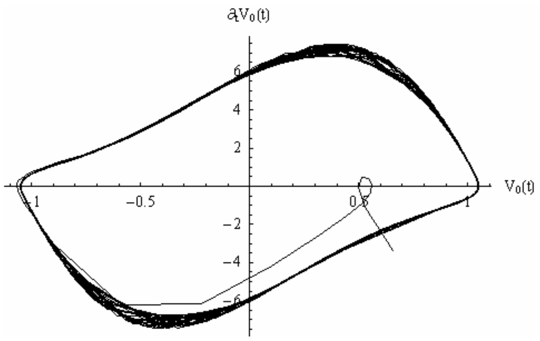


Fig. 2. Phase trajectory of voltage oscillations
for the ordinary Josephson junction

The modes of current oscillations and voltage modes in the Josephson junction depend essentially on the conductivity ratio of a unit of the composite material $\xi$. This fact is clearly seen from the phase portraits matching the voltage oscillations in the Josephson junction for close but different values of the parameter $\xi$ (Fig. 3).

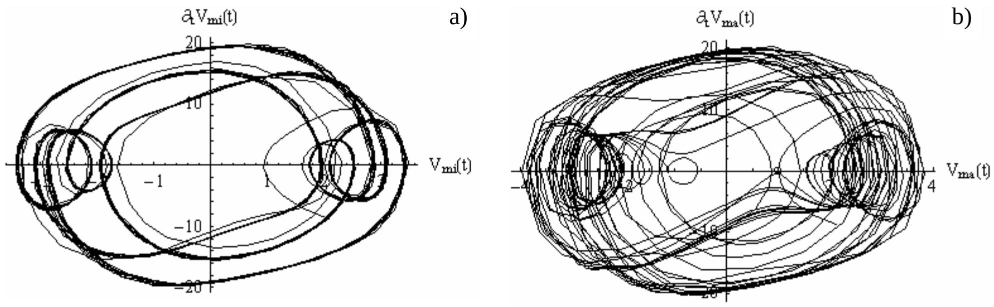


Fig. 3. Phase trajectories of voltage oscillations for the fractal
Josephson junction for $\xi = 0.1$ (a) and $\xi = 0.09$ (b)

As the characteristics of oscillations are very sensitive to modifications of the parameter $\xi$, it is possible to use slow signal modulation (compared to its own junction frequency) to record information on the statistical characteristics of realization in time. Such a modulation can be carried out, for example, by using a slowly changing magnetic field. We now consider the elementary time dependence $\xi(t)$. Let all the intervals of time evolution be split into three equal parts, so that from the onset up to $t = 18$ the value $\xi = 0.1$, then $\xi = 0.09$, and in time interval $36 < t < 50$ again. The realization and phase portrait for the solution of the system equations for such a non-stationary parameter are shown in Figure 4.

It is possible to show that the analysis of the statistical characteristics of the obtained realization by ATD (adaptive testing of device) [5] gives the possibility of restoring the slow evolution of the parameter $\xi$, and hence the possibility of restoring the information signal that affected the modulation.

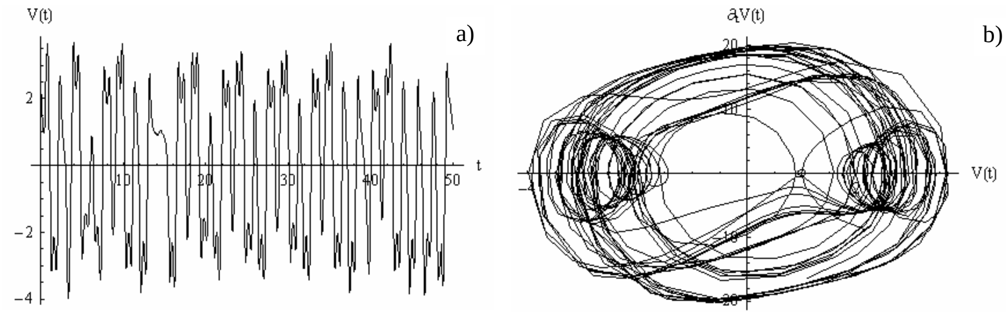


Fig. 4. The realization and phase trajectory of voltage oscillations for the fractal Josephson junction with a non-stationary parameter $\xi$. From the beginning of the evolution up to $t = 18$ the value $\xi = 0.1$, then $\xi = 0.09$, after which in the time interval $36 < t < 50$ again $\xi = 0.1$

## 3. Conclusions

In summary, the Josephson junction with a fractal layer is characterized by nonlinear properties, which permits the oscillation mode to be operated effectively by slowly changing the conductivity ratio of the unit of composite materials.

The Josephson junction with a fractal layer displays even more interesting properties, due to the fact that the fractal medium possesses memory. A more adequate model of the tunnel junction that takes the fractal medium into account can be built from the generalized oscillation equation using fractional operators. A detailed analysis of the model for chaotic communications on the basis of the fractal Josephson junction will be presented in our further investigations.

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