# Resonant tunnelling through a single level quantum dot attached to ferromagnetic leads with non-collinear magnetizations

M. WAWRZYNIAK<sup>1\*</sup>, M. GMITRA<sup>1</sup>, J. BARNAŚ<sup>1,2</sup>

<sup>1</sup>Department of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland

<sup>2</sup>Institute of Molecular Physics, Polish Academy of Sciences, M. Smoluchowskiego 17, 60-179 Poznań, Poland

Resonant tunnelling through a non-interacting single-level quantum dot attached to ferromagnetic leads is analysed theoretically. The magnetic moments of the leads are assumed to be non-collinear. Apart from this, an external magnetic field is applied to the system, which is non-collinear with the magnetizations. The magnetic moments of the leads and the external magnetic field are, however, in a common plane. Basic transport characteristics, including current-voltage curves, differential conductance, and tunnel magnetoresistance associated with magnetization rotation, are calculated using the non-equilibrium Green function technique. The dependence of transport characteristics on the bias voltage has been calculated numerically.

Key words: spin dependent transport; tunnel magnetoresistance; quantum dot

## 1. Introduction

Transport of spin-polarized electrons through quantum dots coupled to ferromagnetic leads is currently of interest due to possible applications in magnetoelectronic and spintronic devices [1–3]. This applies especially to spin valve structures, where the transition from parallel to antiparallel magnetic configuration leads to a large magnetoresistance effect. Additional interesting effects, such as the precession of the dot spin, can arise in structures with a non-collinear orientation of the magnetic moments of external leads [4, 5]. Such a precession results from an effective exchange field exerted on the dot by the ferromagnetic electrodes.

The magnetism of the electrodes can also lead to some other phenomena, for instance the zero bias anomaly in the cotunneling regime [6]. Additional interesting

<sup>\*</sup>Corresponding author, e-mail: lauraphy@spin.amu.edu.pl

effects may arise when an external magnetic field is applied to the dot, leading to a spin splitting of a discrete level and modification of transport properties [7]. In this paper, we consider transport through a non-interacting (U=0) single-level dot (impurity) with non-collinear magnetic moments of the leads. The dot is assumed to be in an external magnetic field, whose orientation is non-collinear with the magnetic moments of the leads. Thus, the results generalize those obtained in Ref. [5].

# 2. The model

The structure under consideration is a non-interacting single-level quantum dot attached to ferromagnetic leads with non-collinear magnetizations. Spin degeneration of the discrete level is removed by an external magnetic field, which is non-collinear with the magnetizations of the electrodes. The field and magnetizations, however, are assumed to be in a common plane. The axis z of the reference frame is assumed to be along the field, whereas the spin moments of the leads are in the (x,z) plane and form angles  $\phi_R$  (right lead) and  $\phi_L$  (left lead) with axis z, as shown in Fig.1.

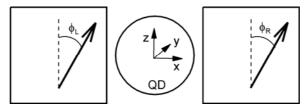


Fig.1. Schematic illustration of the considered system

The whole system is described by the Hamiltonian of a general form

$$H = \sum_{\alpha} H_{\alpha} + H_D + H_T \tag{1}$$

where  $H_{\alpha}$  describes the electrodes

$$H_{\alpha} = \sum_{k} \sum_{\beta = +} \varepsilon_{\alpha k \beta} \, a_{\alpha k \beta}^{\dagger} \, a_{\alpha k \beta} \tag{2}$$

for  $\alpha = L$ , R (left and right electrode, respectively). Here,  $\varepsilon_{\alpha k \beta}$  is the energy of an electron with a wave vector  $\mathbf{k}$  and spin  $\beta$  in electrode  $\alpha$ , whereas  $a_{\alpha k \beta}^+$  and  $a_{\alpha k \beta}$  are creation and annihilation operators, respectively. The electrostatic energy is included in  $\varepsilon_{\alpha k \beta}$ , i.e.  $\varepsilon_{\alpha k \beta} = \varepsilon_{\alpha k \beta}^0 + e U_e^\alpha$ , where e (e < 0) stands for the electron charge,  $\varepsilon_{\alpha k \beta}^0$  is the single-particle energy for the unbiased system, and the electrostatic potentials  $U_e^L = V_t/2$  and  $U_e^R = -V_t/2$  are applied to the system, with  $V_t$  being the transport voltage.

The dot is described by the Hamiltonian

$$H_D = \sum_{\sigma} \varepsilon_d^{\sigma} c_{\sigma}^{\dagger} c_{\sigma} \tag{3}$$

where  $c_{\sigma}^{\dagger}$  and  $c_{\sigma}$  are creation and annihilation operators for spin  $\sigma$ , whereas  $\varepsilon_{d}^{\sigma}$  is the energy level of the dot for  $\sigma = \uparrow$  or  $\sigma = \downarrow$ . The Zeeman splitting of the dot level is  $\Delta_{Z} = \varepsilon_{d}^{\uparrow} - \varepsilon_{d}^{\downarrow}$ . Finally, the tunnelling term  $H_{T}$  in the Hamiltonian in (1) has the form

$$H_{T} = \sum_{\alpha} \sum_{\mathbf{k}} \left\{ \left[ T_{\alpha \mathbf{k}+} a_{\alpha \mathbf{k}+}^{\dagger} \cos \frac{\phi_{\alpha}}{2} - T_{\alpha \mathbf{k}-} a_{\alpha \mathbf{k}-}^{\dagger} \sin \frac{\phi_{\alpha}}{2} \right] c_{\uparrow} + \left[ T_{\alpha \mathbf{k}+} a_{\alpha \mathbf{k}+}^{\dagger} \sin \frac{\phi_{\alpha}}{2} + T_{\alpha \mathbf{k}-} a_{\alpha \mathbf{k}-}^{\dagger} \cos \frac{\phi_{\alpha}}{2} \right] c_{\downarrow} + h.c. \right\}$$

$$(4)$$

where  $T_{\alpha k\beta}$  stands for the appropriate tunnelling matrix element. According to our definition, the spin projection on the local quantisation axis in the leads is denoted by  $\beta = +$  and  $\beta = -$  for spin-majority and spin-minority electrons, respectively, whereas spin projection on the global quantization axis (axis z in this case) is denoted by  $\sigma = \uparrow$  and  $\sigma = \downarrow$ .

### 3. The method

In order to calculate the current and other transport characteristics, we have applied the equation of motion technique for the Green function of the dot,  $G_{\sigma\sigma'} = \langle \langle c_{\sigma} \mid c_{\sigma'}^{\dagger} \rangle \rangle$  and calculated the retarded (advanced) Green functions  $G_{\sigma\sigma'}^{r(a)}(\varepsilon) = G_{\sigma\sigma'}(\varepsilon \pm i\eta)$ 

$$G^{r}(\varepsilon) = \left[ 1 - \mathbf{g}^{r}(\varepsilon) \mathbf{\Sigma}^{r}(\varepsilon) \right]^{-1} \mathbf{g}^{r}(\varepsilon)$$
 (5)

where

$$\mathbf{g}'(\varepsilon) = \begin{pmatrix} (\varepsilon - \varepsilon_d^{\uparrow} + i\eta)^{-1} & 0 \\ 0 & (\varepsilon - \varepsilon_d^{\downarrow} + i\eta)^{-1} \end{pmatrix}$$
 (6)

and

$$\Sigma^{r}(\varepsilon) = \begin{pmatrix} \sum_{0+}^{r}(\varepsilon) & \sum_{1}^{r}(\varepsilon) \\ \sum_{1}^{r}(\varepsilon) & \sum_{0-}^{r}(\varepsilon) \end{pmatrix}$$
 (7)

The self energies are given by the formulae

$$\Sigma_{0\pm}^{r}(\varepsilon) = -\sum_{\alpha} \frac{\Gamma_{0}^{\alpha}}{2} (1 \pm p_{\alpha} \cos \phi_{\alpha}) \left[ \frac{1}{\pi} \ln \left( \frac{B + eU_{e}^{\alpha} - \varepsilon}{B - eU_{e}^{\alpha} + \varepsilon} \right) + i \right]$$
 (8)

$$\Sigma_{1}^{r}(\varepsilon) = -\sum_{\alpha} \frac{\Gamma_{0}^{\alpha}}{2} (p_{\alpha} \sin \phi_{\alpha}) \left[ \frac{1}{\pi} \ln \left( \frac{B + eU_{e}^{\alpha} - \varepsilon}{B - eU_{e}^{\alpha} + \varepsilon} \right) + i \right]$$
 (9)

where the electron band is assumed to extend from -B to B,  $p_{\alpha}$  is the spin polarization of the  $\alpha$ -th electrode, and  $\Gamma_0^{\alpha}$  is a constant defined via the relation  $\Gamma_{\pm}^{\alpha}(\varepsilon) = \Gamma_{\pm}^{\alpha} = \Gamma_0^{\alpha}(1 \pm p_{\alpha})$ , where  $\Gamma_{\pm}^{\alpha}(\varepsilon) = 2\pi \sum_{\mathbf{k}} |T_{\alpha\mathbf{k}\pm}|^2 \delta(\varepsilon - \varepsilon_{\alpha\mathbf{k}\pm})$  is assumed to be

independent of the energy within the electron band. In the following, we assume that  $\Gamma_0^L = \Gamma_0^R = \Gamma_0$ .

The next step is to obtain the lesser Green function from the formula  $G^{<}(\varepsilon) = G^{r}(\varepsilon)\Sigma^{<}(\varepsilon)G^{a}(\varepsilon)$ . Here,  $\Sigma^{<}(\varepsilon) = -\sum_{\alpha} [\Sigma_{\alpha}^{r}(\varepsilon) - \Sigma_{\alpha}^{a}(\varepsilon)]f_{\alpha}(\varepsilon)$  and  $f_{\alpha}(\varepsilon)$  is

the Fermi-Dirac distribution function for lead  $\alpha$ ,  $f_{\alpha}(\varepsilon) = 1/\{1 + \exp[(\varepsilon - eU_e^{\alpha})/k_BT]\}..$ 

The average occupation numbers can be calculated from the formula [5]

$$\langle n_{\sigma} \rangle = \langle c_{\sigma}^{\dagger} c_{\sigma} \rangle = \operatorname{Im} \int_{-\infty}^{+\infty} \frac{d\varepsilon}{2\pi} G_{\sigma\sigma}^{<}(\varepsilon)$$
 (10)

and

$$\langle n_{\sigma-\sigma}\rangle = \langle c_{\sigma}^{\dagger}c_{-\sigma}\rangle = -i\int_{-\infty}^{+\infty} \frac{d\varepsilon}{2\pi} G_{-\sigma\sigma}^{<}(\varepsilon),$$
 (11)

whereas the electric current flowing from the  $\alpha$ -th electrode to the dot is given by the general expression

$$J_{\alpha} = \frac{ie}{\hbar} \int_{-\infty}^{+\infty} \frac{d\varepsilon}{2\pi} \operatorname{Tr} \left\{ \mathbf{\Gamma}_{\alpha} \left[ \mathbf{G}^{<}(\varepsilon) + f_{\alpha}(\varepsilon) \left( \mathbf{G}^{r}(\varepsilon) - \mathbf{G}^{a}(\varepsilon) \right) \right] \right\}$$
(12)

with

$$\Gamma_{\alpha} = \Gamma_{0} \begin{pmatrix} 1 + p_{\alpha} \cos \phi_{\alpha} & p_{\alpha} \sin \phi_{\alpha} \\ p_{\alpha} \sin \phi_{\alpha} & 1 - p_{\alpha} \cos \phi_{\alpha} \end{pmatrix}$$
(13)

The current J flowing through the system must be conserved,  $J_L = -J_R$ , so one can also calculate J as  $J = (1/2)(J_L - J_R)$ .

### 4. Numerical results and discussion

In this section, we discuss some numerical results for non-linear transport, assuming an empty dot level at equilibrium,  $\varepsilon_d = 0.25$  eV. The electric current for the paral-

lel configuration,  $\phi_L - \phi_R = 0$ , is shown in Fig. 2a. The dashed line represents the total current in the absence of a magnetic field,  $\Delta_Z = 0$ . The solid line corresponds to the case with Zeeman splitting  $\Delta_Z = 0.05$  eV, while the dotted and dashed-dotted lines represent the corresponding contributions from two spin channels. The inset in Fig. 2a describes the case with  $\Delta_Z = 0.05$  eV, and shows the suppression of current with increasing angle between the spin polarizations of the electrodes.

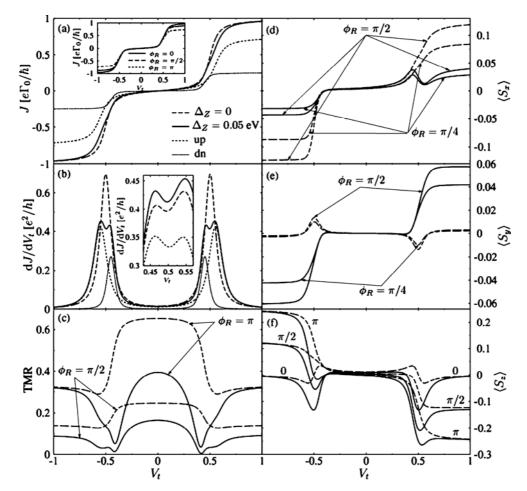


Fig. 2. Transport characteristics versus bias voltage  $V_t$  for  $\phi_L = 0$ . The other parameters are T = 100 K,  $\Gamma_0 = 0.02$  eV,  $\varepsilon_d = 0.25$  eV, and the electron band extends from -3.3 to 3.3 eV: a) the bias dependence of electric current in the parallel configuration,  $\phi_R = 0$ . The dashed line corresponds to the case with no Zeeman splitting,  $\Delta_Z = 0$ . The solid line refers to  $\Delta_Z = 0.05$  eV, with the corresponding contributions of the two spin channels represented by the dotted and dashed-dotted lines. The inset shows current—voltage curves for  $\Delta_Z = 0.05$  eV and for  $\phi_R = 0$ ,  $\phi_R = \pi/2$  and  $\phi_R = \pi$ ; b) differential conductance corresponding to the current—voltage curves shown in a. The inset shows the differential conductance for the same values of  $\phi_R$  as in the inset in a; c) the magnitude of TMR for  $\Delta_Z = 0.05$  eV (solid lines) and  $\Delta_Z = 0$  (dashed lines); parts (d), (e), and (f) present the bias dependences of the average spin components,  $\langle S_x \rangle$ ,  $\langle S_y \rangle$  and  $\langle S_z \rangle$ , for  $\Delta_Z = 0.05$  eV (solid lines) and  $\Delta_Z = 0$  (dashed lines)

In all cases, a significant current starts to flow when the discrete level of the dot crosses the Fermi level of the source electrode. The steps in the electric current lead to peaks in differential conductance, as shown in Fig. 1b for the same situations as in Fig. 1a. The peaks in external magnetic field (solid line) are spin-split and their amplitude becomes smaller compared to that for  $\Delta_Z=0$  (dashed line). This splitting is a consequence of the Zeeman splitting of the discrete dot level. The inset in Fig. 1b shows that conductance decreases with increasing angle between the spin polarizations of the electrodes.

The magnitude of the tunnel magnetoresistance (TMR) effect, defined as  $[J(\phi_R = 0, \phi_L = 0) - J(\phi_R, \phi_L)]/J(\phi_R = 0, \phi_L = 0)$ , is shown in Fig. 2c for  $\phi_L = 0$  and two values of the angle  $\phi_R$ . In general, the TMR effect increases with increasing angle between spin polarizations. We also notice that the TMR effect is larger below the threshold voltage, where current flows mainly due to higher-order tunnelling processes [6].

The right column of Fig. 2 (parts d-f) shows the bias dependence of the average dot spin components. The components are derived from Eqs. (10) and (11) and are given in units of  $\hbar$  by the formulas  $\langle S_x \rangle = \text{Re}\{n_{\uparrow\downarrow}\}$ ,  $\langle S_y \rangle = \text{Im}\{n_{\uparrow\downarrow}\}$ , and  $\langle S_z \rangle = \langle n_{\uparrow} - n_{\downarrow} \rangle/2$ . The average values  $\langle S_x \rangle$  and  $\langle S_y \rangle$  are zero for any bias voltage in the parallel and antiparallel configurations, and are not plotted in Fig. 2. It is worth noting that an external magnetic field removes the antisymmetrical behaviour of  $\langle S_z \rangle$  for the antiparallel configuration. The  $\langle S_z \rangle$  component is symmetrical with respect to the bias voltage only in the parallel configuration. Figure 2e shows the y-component of the average spin. A nonzero value of this component indicates spin precession due to interaction between the dot and electrodes.

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