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Persistent currents controlled by non-classical electromagnetic fields

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Mesoscopic systems and non-classical electromagnetic fields are of central importance to quantum information processing. Our aim is to present the significant influence of non-classical radiation on the properties of persistent currents. We study mesoscopic rings subject to both classical and non-classical electromagnetic fields. Our discussion is limited to one- and two-mode fields prepared in a given quantum state. We show that non-classical fields with a definite phase can induce persistent currents even in the absence of classical driving. There are two qualitatively distinct classes of two-mode electromagnetic fields: separable and entangled. This is reflected in the properties of the current, which becomes time-dependent for fields in an entangled state. We extend our earlier work and investigate the effect of entanglement for a family of various states with various amounts of entanglement: from separable states to Bell states quantified by concurrence.

Key words: *persistent current; non-classical electromagnetic field; entangled state*

Mesoscopic devices operating at temperatures 0.1–1 K exhibit interesting quantum phenomena. Most work involves the interaction of these devices with classical microwaves and static magnetic fields. In our recent papers [1, 2], we have investigated the properties of persistent currents [3] in mesoscopic normal metal rings or cylinders subject to both classical and non-classical electromagnetic fields prepared in a given quantum state. The emphasis in these studies is on the properties arising from the quantum nature of the electromagnetic field and cannot be understood classically. The present work shows how one- and two-mode electromagnetic fields prepared in some special quantum states influence persistent currents. Persistent currents in mesoscopic metallic and semiconducting rings at low temperature are a signature of electron phase coherence [3]. They can be induced by a static magnetic flux φ_e at $T < T^*$, where $T^* = \Delta/(2\pi^2)$ and Δ is the quantum size energy gap. The formula for the persistent current is ($\hbar = c = k_B = 1$) [4]:

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$$I_e\left(\frac{\varphi_e}{\varphi_0},T\right)=I_0\sum_{n=1}^{\infty}A_n(T)\sin\frac{2\pi n\varphi_e}{\varphi_0}$$
$$A_n(T)=\frac{4T}{\pi T^*}\frac{\exp(-nT/T^*)}{1-\exp(-2nT/T^*)}\cos(nk_F l_x) \tag{1}$$

where $\varphi_0 = 2\pi/e$ is the flux quantum, I_0 is the current amplitude, and k_F is the Fermi wave vector. The currents are periodic functions of φ_e/φ_0 and depend on certain parameters (e.g. radius of the ring). They can be paramagnetic or diamagnetic at small φ_e. The following discussion is limited to the case of paramagnetic currents flowing in quasi 1D rings with an even number of coherent electrons. The result for diamagnetic currents can be obtained if one replaces φ_e by $\varphi_e + \varphi_0/2$ [1]. For quantum electromagnetic fields, the electric and magnetic fields are the well-known dual quantum variables [6]. One can also associate them, by simple integration around the circumference of the ring, with variables representing electromotive force and the magnetic flux. The operator of magnetic flux evolves in the Heisenberg picture, and after suitable renormalization, in the following way (for details see [5]):

$$\varphi(t)=\frac{1}{\sqrt{2}}\left[\exp(i\omega t)a^+ + \text{h.c.}\right]$$

In further considerations, we assume that the energy scales $T < \omega < \Delta$ are well separated in order to satisfy "adiabatic" conditions [1] and avoid various non-linear effects. Since the coherent current depends on the ratio of the flux and flux quantum φ_0 rather than the flux itself, we introduce the re-scaled flux operator

$$x=\frac{\varphi_e}{\varphi_0}+\frac{\varphi}{\varphi_0}\equiv\lambda+x_q \tag{2}$$

where λ is the classical magnetic flux (c-number) and x_q is the operator of the non-classical flux (both in φ_0 units). As a result of the presence of non-classical fields, the current itself is no longer a c-number, but becomes a quantum mechanical operator. This current operator, rescaled with respect to the current unit I_0, reads after rearrangement (for details see Ref. [1]):

$$\begin{aligned}I_c &:= \sum_{n=1}^{\infty}A_n(T)\exp(i2\pi nx)=\sum_{n=1}^{\infty}A_n(T)\exp(i2\pi n\lambda)\exp(i2\pi nx_q)\\ &=\sum_{n=1}^{\infty}A_n(T)\exp(i2\pi n\lambda)D(n\xi e^{i\omega t})\end{aligned} \tag{3}$$

where $D(A)$:= $\exp(Aa^+ - A^*a)$ is the displacement operator [6], $\xi=\sqrt{2}\pi/\varphi_0$, and ρ is the density operator of the electromagnetic field. The observable quantity is the imaginary part of the expectation value of the current operator:

$$I(x,T) = \Im m\langle I_c\rangle = \Im m Tr(I_c\rho) = \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi n\lambda)W(\varsigma_n) \tag{4}$$

and the calculation of the expectation value of the current reduces to the calculation of the Weyl function $W(\zeta_n) = Tr(\rho D(n\xi\exp(i\omega t)))$, with $\zeta_n = n\xi\exp(i\omega t)$. We see that persistent currents calculated from Eq. (4) are influenced by the features of non-classical radiation contained in the Weyl function.

The most popular "object" of quantum computing is the qubit described by:

$$\rho = |\psi\rangle\langle\psi|;|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i\delta}}{\sqrt{2}}|1\rangle \tag{5}$$

where the phase $\delta \in \langle 0, 2\pi)$ reflects the different superpositions of the two lowest number eigenstates of the non-classical field. The qubit is a quantum mechanical analogue of the bit and is subject to intensive research itself. In our case, it is the most general state of the electromagnetic field, involving only the two lowest energy eigenstates. In the state described by Eq. (5), the number of photons is not definite and therefore the state has a definite phase (the phase and the number of photons are dual variables [7]). We have already found [1] that the persistent current calculated in a state with a phase is time dependent.

In order to take into account the noise present in every real experiment, we assume that the qubit, before reaching the ring, is transmitted via the depolarizing channel [8]:

$$\rho' = \Pi(\rho) = \frac{I_d}{2} + (1-p)\rho \tag{6}$$

where $I_d/2$ (given by the identity matrix I_d [8]) is the maximally mixed state of the two-state system, and p describes a classical probability.

The behaviour of the current calculated with ρ given by Eq. (5) is plotted in Figs. 1 and 2, for which two important effects can be seen. The first is the lowering of the overall amplitude of the current with increasing p. It is shown in Fig. 1, where the current, being a function of the classical flux λ, is plotted for several values of p for $\delta = 0$ and $t = 0$. This effect does not kill the current even for $p = 1$.

The second effect is connected to the phase δ. We find that a choice of the phase leads to a finite value of the current, even in the absence of the classical flux ($\lambda = 0$). This is shown in Fig. 2, where, in the main graph, the current $I(\lambda = 0)$ vs. δ is plotted at $t = 0$. We see that non-classical electromagnetic fields with different values of δ can produce, in contrast to the classical flux, both diamagnetic and paramagnetic currents. Note that for the state with $p = 1$, the current $I(\lambda = 0) = 0$. The inset in Fig. 2 shows the time dependence of the current $I(\lambda = 0)$ for two values of δ. A similar effect has already been discovered for coherent states in Ref. [1].

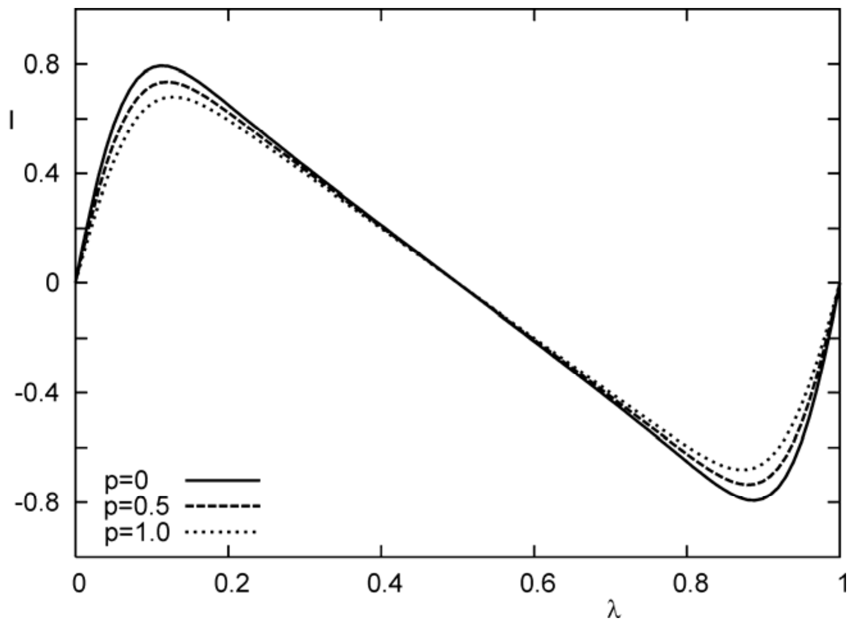

Fig. 1. Persistent current vs. classical flux in the presence of a depolarised qubit for three values of p and $\delta = t = 0$

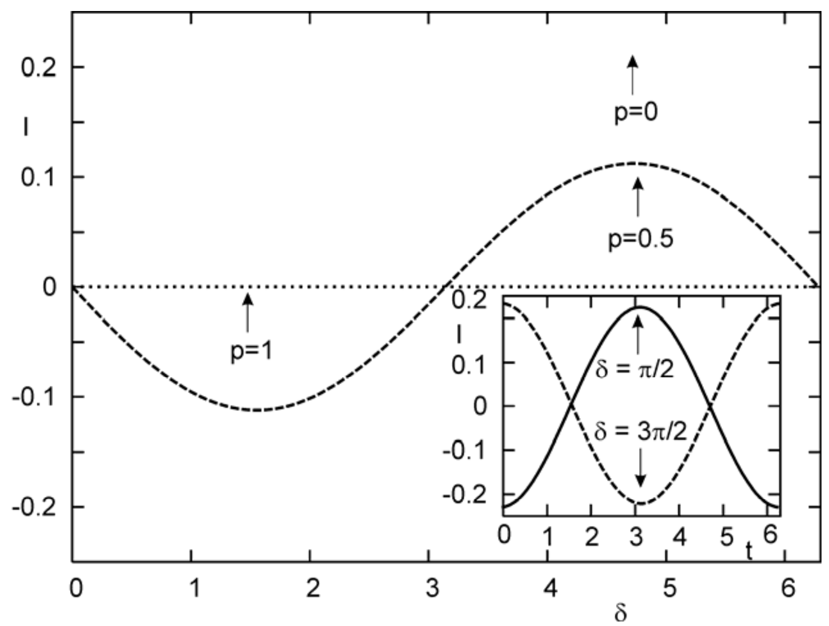

Fig. 2. Current in the presence of a depolarised qubit vs. the phase δ in the absence of classical flux $\lambda = 0$ for different values of p and $t = 0$ (main graph). The inset shows the time dependence of the current $I(\lambda = 0)$ for two values of δ and $p = 0$

Entangled states of photons are of central interest in quantum communication and quantum information. Their "classically unusual" properties allow one to expect that the current in the presence of such states will also become "unusual". The expectation value of the complex current in the mesoscopic ring in the presence of two-mode fields with frequencies ω_1 and ω_2 is given by [1]

$$\langle I_c \rangle = \sum_{n=1}^{\infty} A_n(T)\exp(i2\pi n\lambda) W(\varsigma_{1n}, \varsigma_{2n}) \tag{7}$$

where the two-mode Weyl function is

$$W(\varsigma_{1n}, \varsigma_{2n}) = Tr\left(\rho D_1\left(n\xi \exp\left(i\omega_1 t\right)\right) D_2\left(n\xi \exp\left(i\omega_2 t\right)\right)\right)$$

with $\varsigma_{in} = n\xi \exp(i\omega_i t)$.

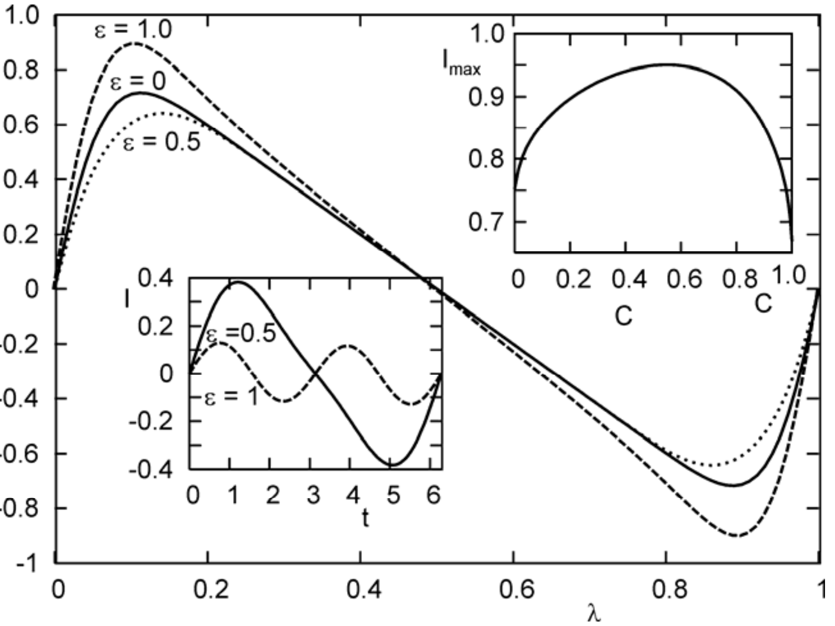

Fig. 3. Persistent currents in the presence of two-mode fields with different amounts of entanglement at $t = 0$. Inset 1 (bottom): time dependence of the current for $\lambda = 0$. Inset 2 (top): amplitude of the current as a function of concurrence C

In the following, we investigate the properties of the currents in the presence of the following family of states

$$|\psi\rangle = \frac{1}{\sqrt{2}}(a(\varepsilon)|00\rangle + b(\varepsilon)|01\rangle + c(\varepsilon)|10\rangle + d(\varepsilon)|11\rangle \tag{8}$$

with

$$a(\varepsilon)=\sqrt{2}-\varepsilon\left(1-\sqrt{2}\right), \quad d\left(\varepsilon\right)=\varepsilon,$$
$$b(\varepsilon)=\sqrt{2-a^2(\varepsilon)+d^2(\varepsilon)} \quad \text{and} \quad c(\varepsilon)=0 \quad \text{for} \quad 0\le\varepsilon\le 1$$

The amount of entanglement in this family of states increases with ε and can be measured by the concurrence $C = |ab|$ [9]. The problem of time-dependence has already been discussed in Ref. [2]. Here, we focus on the amplitude of the current. The plot of the resulting current vs. λ for $\omega_1 \approx \omega_2$ is given in Fig. 3. Notice that both the overall amplitude (main graph) and the current in the absence of classical flux ($\lambda = 0$) approach their maxima for non-extremum values of ε, namely $\varepsilon \approx 0.5$ (the upper inset in Fig. 3). Furthermore, the presence of entangled light also changes the period of the time dependence of the current (the lower inset in Fig. 3).

It is well known that persistent currents can be driven by static magnetic fluxes [3]. In this paper, we show that they can also be induced by non-classical electromagnetic fields. The purpose of these considerations is to present some interdisciplinary research that exploits the quantum nature of the non-classical fields, which are studied in optics in order to control the behaviour of mesoscopic quantum devices. Persistent currents flowing in mesoscopic rings or cylinders are placed on the border of the classical and quantum worlds. Hence, as expected, they can be controlled by both classical and quantum parameters. In this paper, we have investigated the possibility of quantum-like control. We have shown that one obtains the time-dependence of the current, provided that the non-classical field is in a state with a well-defined phase (indefinite number of photons). Such a field can produce a current even in the absence of classical driving. Further, the effect of entanglement in a two-mode electromagnetic field was considered. We have studied fields with different amounts of entanglement as measured by concurrence, and found that a moderate (non-extremum) amount of entanglement can enhance the current amplitude relative to the case of separable (ε =0) fields. The work belongs to the general context of studying fully quantum mechanical devices comprising mesoscopic devices and non-classical electromagnetic fields. Such devices are potentially useful for quantum technologies and quantum information processing. The presented results show that persistent currents can serve as detectors of non-classical properties of radiation or as efficient tools in quantum-driving mesoscopic systems.

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