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# Current-induced torque in ferromagnetic single-electron devices in the limits of the fast and slow spin relaxation

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Theoretical analysis of the spin-transfer torque acting on the magnetic moment of the central electrode (island) in a single-electron ferromagnetic transistor has been performed for the spin relaxation time in the island ranging from fast to slow spin relaxation limits. The magnetic configuration of the system can be generally arbitrary. Spin accumulation on the island, due to the spin asymmetry of tunnelling processes, is taken into account. Electric current flowing through the device is calculated in the regime of sequential transport, and the master equation is used to calculate probabilities of different charge and spin states in the island. The torque acting on the central electrode is then calculated from the spin current absorbed by magnetic moment of the island.

Key words: *ferromagnetic single-electron transistor, spin-polarized current, spin-transfer torque*

## 1. Introduction

Despite preliminary theoretical predictions that spin transfer in tunnel junctions could be imperceptible [1], mainly due to the much smaller current density than in the case of metallic devices, current-driven magnetic switching in tunnel junctions has been found [2, 3]. Motivated by recent theoretical and experimental findings, we present calculations of the spin-transfer torque in ferromagnetic double-barrier junctions with Coulomb blockade effects. Spin accumulation in the central electrode due to the spin asymmetry of tunnelling processes is taken into account. The electric current flowing through the system is calculated in the regime of sequential transport. The master equation is set up to determine the relevant occupation probabilities of differ-

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ent charge and spin states in the island. The torque acting on the central electrode is then calculated from the spin current absorbed by the magnetic moment of the island.

## 2. Model and method

The system considered in this paper is presented in Fig. 1. It consists of three ferromagnetic electrodes – the left ($l$), right ($r$) and the middle one ($i$) referred to as an island. The magnetic configuration of the system can be generally non-collinear. The vectors $\vec{S}_l$ ($\vec{S}_r$) and $\vec{S}_i$ indicate the net spin moments of the left (right) electrode and of the island, respectively. As indicated in the figure, the net spin of each electrode can form an angle with the net spin of the island. There is also a bias voltage applied to the system, $V = V_l - V_r$, where $V_l$ and $V_r$ are electrostatic potentials of the left and right electrodes. Apart from this, the island is capacitively coupled to a gate with the corresponding voltage $V_g$.

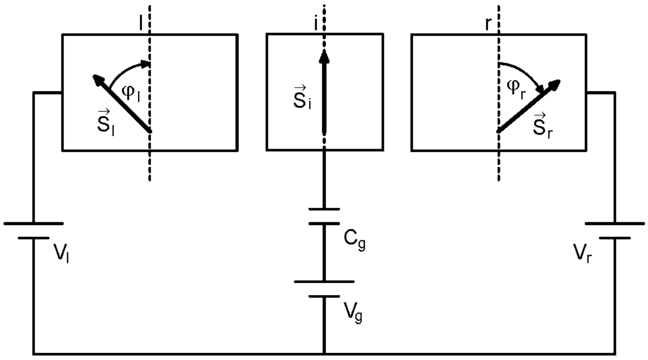


Fig. 1. A schematic diagram of the ferromagnetic single-electron transistor with all the electrodes being ferromagnetic. The vectors $\vec{S}_l$, $\vec{S}_r$ and $\vec{S}_i$ indicate the net spin moments of the external electrodes and of the island, respectively. The angle between the net spin of the left (right) electrode and the net spin of the island is denoted by $\varphi_r$ and $\varphi_l$. There is a bias voltage applied to the left and right electrodes, while the island is capacitively coupled to a gate voltage $V_g$

We assume that the island is large enough so that the level quantization can be neglected but, on the other hand, sufficiently small to have the charging energy $e^2/2C$ significantly larger than the thermal energy $k_BT$, where $C$ is the total island capacitance, $e$ is the electronic charge, $k_B$ denotes the Boltzmann constant, while $T$ stands for temperature. If this condition is fulfilled, the electrons tunnel through the system one by one, giving rise to a flowing current. Furthermore, the system exhibits the so-called single-electron charging effects, such as, for example, blockade of the current below a certain threshold voltage (Coulomb blockade), the step-like $I$–$V$ characteristics (Coulomb staircase), etc. [4] In addition, if the electrodes are ferromagnetic, further interesting effects arise due to the interplay of single-charge tunnelling and ferromagnetism. They include the oscillations of tunnel magnetoresistance when sweeping the bias voltage, spin accumulation, etc. [5, 6].

In order to calculate transport properties, the two-channel model is applied; the current flows through the system due to consecutive tunnelling events in the spin-majority and spin-minority channels. Each tunnel junction is characterized by its resistance for the spin-majority and spin-minority electrons, and by its capacitance. In this analysis, we assume that the total resistance of each tunnel barrier $R_k$ ($k = l, r$) is much larger than the quantum resistance $R_q = h/e^2$. This condition implies that the charge on the island is localized well and the orthodox theory [4] is applicable. Within this theory, only the sequential tunnelling processes are taken into account, while the corresponding rates are given by the Fermi golden rule. Having determined all the possible tunnelling rates, one can set up a master equation to calculate the probabilities that there is a given number of extra electrons on the island. The current flowing through the system can be then calculated from the appropriate equations, as described for example by Amman et al. [7].

Moreover, in this analysis we assume that the energy relaxation time is much shorter than the time between two successive tunnelling events, while the spin relaxation time in the island, $\tau_{sf}$, may be arbitrary. If the spin relaxation time is longer than the time between two tunnelling processes, there is a nonequilibrium spin accumulation induced in the island. Because the island is ferromagnetic, the shifts of the Fermi level for the spin-majority and spin-minority bands due to spin accumulation are not equal. The ratio of the Fermi level shifts for the corresponding subbands is determined by the ratio of the density of states for a given electron subband. Therefore, to calculate the shift of the Fermi level for an arbitrary spin relaxation time $\tau_{sf}$, the following balance equation [5, 6, 8] should be solved self-consistently:

$$\left(I_r^\sigma - I_l^\sigma\right) - \frac{D_i \Omega_i}{\tau_{sf}} \Delta E^\sigma = 0$$

Here, $I_r^\sigma$ and $I_l^\sigma$ are the currents flowing through the right and left junctions, respectively, in the spin-majority ($\sigma = \uparrow$) or spin-minority ($\sigma = \downarrow$) channels, $D_i$ indicates the density of states of the island, while $\Omega_i$ is the island volume. The shifts of the Fermi level for the spin-majority or spin-minority electrons are denoted by $\Delta E^\sigma$, where $\sigma = \uparrow\downarrow$.

Due to the spin asymmetry of ferromagnetic electrodes, the currents flowing in the spin-majority and spin-minority channels are different. Furthermore, if there is a certain angle between the net spin of the island and each of the electrodes, a single electron when tunnelling from one electrode to the island adjusts its spin orientation in an interfacial layer of atomic thickness. As a consequence, some angular momentum is transmitted to the local magnetization of the island and the electrode, producing spin torque. The torque acting on the magnetic moment of the island can be calculated from the difference between the spin current flowing into the island and the spin current leaving the island, in a similar way as presented in the papers by Slonczewski

[9, 10]. In the case of our system, the total torque $\tau_i$ acting on the net spin moment of the central electrode can be determined from the following equation:

$$\tau_i = \sum_{k=l,r} \left(\Delta I_k - \Delta I_{i(k)} \cos\varphi_k\right) \frac{1}{\sin\varphi_k}$$

where $\Delta I_k = I_k^+ - I_k^-$ and $\Delta I_{i(k)} = I_{i(k)}^{\uparrow} - I_{i(k)}^{\downarrow}$ for $k = l, r$. $I_k^+$, $I_k^-$ denote the currents in the spin-majority and spin-minority channels taken at the atomic distance from the barrier for a given electrode $k$, whereas $I_{i(k)}^+$, $I_{i(k)}^-$ are the currents in the spin-majority and spin-minority channels in the island close to the barrier between the island and the $k$-th electrode.

## 3. Numerical results and discussion

In this section, we present numerical results for ferromagnetic single-electron transistors calculated for an arbitrary magnetic configuration of the system and for arbitrary spin relaxation time in the island. The angular dependences of the Fermi level shifts for the spin-majority and spin-minority electrons for several values of the spin relaxation time is presented in Fig. 2.

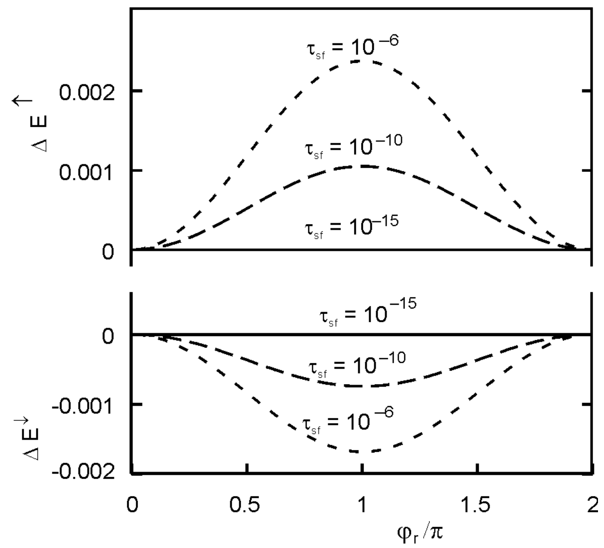


Fig. 2. The shift of the Fermi energy for spin-majority ($\Delta E^{\uparrow}$) (upper part) and spin-minority (($\Delta E^{\downarrow}$) (lower part) electrons due to spin accumulation for various spin relaxation times $\tau_{sf}$ as a function of $\varphi_r$, where $\varphi_l$ =0. The other parameters are: $R_l^{\uparrow p} = 0.3$ MΩ, $R_l^{\downarrow p} = 0.15$ MΩ, $R_r^{\uparrow p} = 5$ MΩ, $R_l^{\downarrow p} = 2.5$ MΩ, where for $k = l, r$ $R_k^{\uparrow ap} = R_k^{\downarrow ap} = (R_k^{\uparrow p} R_k^{\downarrow p})^{1/2}$, $C_l = C_r = C_g = 1$ aF, $V_l = 0.5$ V, $V_r = 0$ V, $V_g = 0$ V, $D_i\Omega_i = 1000$ (eV)$^{-1}$ and $T = 4{,}2$ K

The time between two successive tunnelling events can be estimated to be of the order of $10^{-12}$–$10^{-10}$ s. Consequently, in the case of $\tau_{sf} = 10^{-15}$ s (solid line in Fig. 2) there is no spin accumulation. This is because the spin relaxation time is much shorter than the time between consecutive tunnelling events and the electron spin relaxes before the next tunnelling events occur. However, if $\tau_{sf} = 10^{-10}$ s, the spin relaxation time becomes of the order of the time between successive tunnelling events and there appears a non-zero shift of the Fermi level (see the dashed line in Fig. 2). On the other hand, the third case when $\tau_{sf} = 10^{-6}$ s corresponds to the limit of long spin relaxation and spin accumulation is much enhanced as compared to the two previous cases, see the dotted line in Fig. 2. The upper (lower) part of Fig. 2 shows the shift of the Fermi level for the spin-majority (spin-minority) electrons. It is also worth noting that generally $\Delta E^{\uparrow} > \Delta E^{\downarrow}$, due to the asymmetry in the densities of states of the respective electron bands. Furthermore, as shown in Fig. 2, the maximum spin accumulation occurs for $\varphi_r = \pi$, which corresponds to the antiparallel configuration.

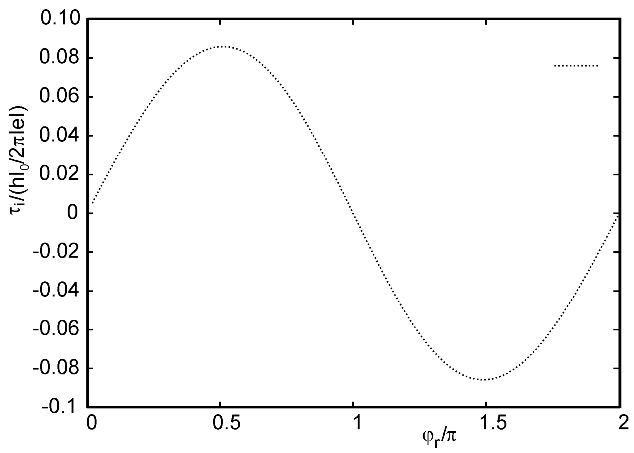


Fig. 3. The normalized torque acting on the island due to the spin-polarized current as a function of $\varphi_r$ calculated for $\varphi_l = 0$ and for different spin relaxation times. The other parameters are the same as in Fig. 1

In Figure 3, we present the normalized torque acting on the central electrode due to the spin polarized current as a function of $\varphi_r$ (while $\varphi_l = 0$) for three different spin relaxation times. As previously, the three values of the spin relaxation time correspond to the limits of fast and slow spin relaxation, and the crossover between those two limits, correspondingly. The normalized torque is defined as $\tau/\hbar I_0/|e|$, where $I_0$ is the current flowing through the system at a constant bias voltage. First of all, one can see that the dependence of the normalized torque on the angle between the right electrode and the island resembles the sine function. Thus, by changing $\varphi_r$, it is possible to produce either positive or negative torque acting on the island. Furthermore, the maximum torque occurs for $\varphi_r = \pi/2$. The normalized torque is independent of the spin relaxation time. This fact can be understood by realizing that both the current and torque apparently depend on $\tau_{sf}$, the normalized torque, however, given by the ratio of the torque and the current, does not.

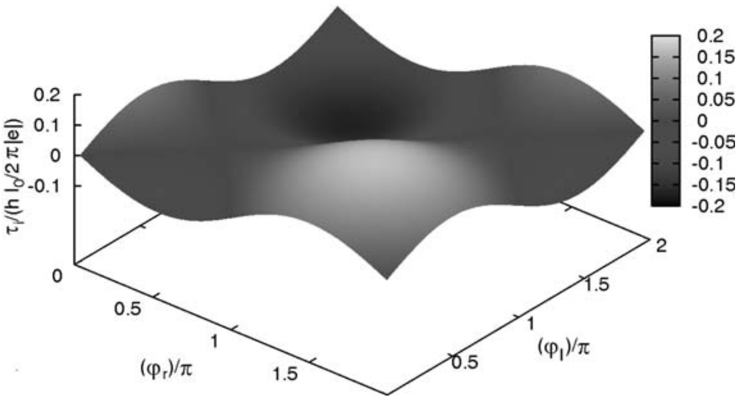


Fig. 4. The normalized torque acting on the island due to the spin-polarized current as a function of $\varphi_r$ and $\varphi_l$ for spin relaxation time $\tau_{sf} = 10^{-6}$ s. The other parameters are the same as in Fig. 1

The dependence of the normalized torque on both angles $\varphi_r$ and $\varphi_l$ is presented in Fig. 4. This figure was calculated for the spin relaxation time corresponding to the limit of slow spin relaxation. However, because the normalized torque only slightly depends on spin relaxation, see Fig. 3, the numerical results presented in this figure can be considered as the dependence of normalized torque on arbitrary values of spin relaxation time. From Fig. 4 one can easily identify the regions where the normalized torque is negative and positive, as well as the regions where the maximum torque occurs.

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