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Transport in nanostructures. Recent developments

K. I. WYSOKIŃSKI*

Institute of Physics and Nanotechnology Centre, M. Curie-Skłodowska University,
ul. Radziszewskiego 10, 20-031 Lublin, Poland

The recent work on the transport through nanostructures is discussed. It turns out that such structures very often directly serve as monitoring and/or controlling devices. They enable us to study transport properties and many body effects in highly controlled conditions. The simplest configuration discussed in the paper consists of a quantum dot (QD) connected to two external electrodes *via* tunnel barriers. Another important goal of the recent studies is to use the electron spin instead of charge in modern electronic devices. For the realization of electronics with spins (called spintronics), a precise control and efficient monitoring of spins is necessary. One way of achieving the goal that is briefly discussed in the paper is by means of the electric field in the presence of spin-orbit coupling *via* the so-called spin Hall effect (SHE).

Key words: *quantum transport; nanostructure; spin Hall effect*

1. Introduction

In this paper, I will concentrate on two aspects related to the general subject of transport in nanostructures. These are: (i) the Kondo effect as an example of many body interaction effects in transport through quantum dot based devices and (ii) the spin Hall effect. The properties discussed are in one way or another related to the spin of electrons and are or may be of importance for the development of spin-based electronics (spintronics) [1].

Theoretical and experimental studies of nanostructures, i.e. systems of a few nanometres dimensions face some common problems such as: (a) discreetness of the spectrum, (b) large charging energies and (c) geometrical and other asymmetries of the structure.

*E-mail: karol@tytan.umcs.lublin.pl

Due to a small size of the structure, its energy spectrum is discrete with typical distances between energy levels ΔE ranging from a fraction of to few millielectronvolts and can be observed even at room temperatures. The small size of the devices also means small capacitance C and large charging energy $E_C = e^2/2C$, where e is the electron charge. Contacts between various parts of the devices are realised *via* tunnelling with controlled tunnel resistance. This allows study of various regimes of electron transport with the Coulomb blockade phenomenon being an important example.

As an example of the many body effects induced by large charging energy, which in the language of interacting systems means large on-site Coulomb repulsion usually known in the solid state community as Hubbard U, the Kondo effect is presented [2] and its influence on some thermoelectric phenomena including conductivity and thermoelectric power of a quantum dot connected to non-magnetic and ferromagnetic leads. It is important to realise that many body effects may have quite an unexpected influence on the behaviour and properties of nanostructures. For example, it has been recently proposed theoretically [3] that the charges detected in the noise of backscattered currents in the system showing Kondo effect should possess values e^* differing from electron charge e. Fractional values of charge of quasi-particles is a property of elementary excitations of two-dimensional electron gas placed in strong perpendicular magnetic field. They have been observed in tunnelling studies of fractional quantum Hall effect [4] with $e^* = e/3$ and in superconductors where $e^* = 2e$. Unlike quantum Hall effect or superconductors, where e^* has a meaning of quasiparticle charge, in the Kondo regime the effective charge is a result of inelastic processes connected with interactions. This observation is potentially important in view of recent proposals to use devices measuring electron charge and based on quantum dots in nano-metrology applications [5].

One of the most important ideas of the recent years is to use quantum mechanical principles for information storage and processing. One of the proposals is to make use of the spin degrees of freedom in quantum dots. This requires coherent control and manipulation of spins. There exist a huge activity, both theoretical and experimental, connected with various proposals to control spins via electrical means. As one example of the technique of spin control and manipulation, we discuss the spin Hall effect.

2. The Kondo effect in transport via a quantum dot

Quantum dots (QD) are small islands containing finite number of charges. If connected to external electrodes, they have been proposed as building blocks of single electron transistors [6], quantum bits or registers of future computers [7] working according to quantum logic, as efficient factories of entangled states [8], precise ampere (current) meters [5] or potentially efficient energy conversion instruments [9]. Because of their small size, the charging energy in the considered structures is large and gives rise to many interesting phenomena. The Kondo effect [10] appearing in quantum dots weakly coupled to external electrodes at low temperatures is one of them. In quantum dots, it has been studied in various geometries. The dots attached to normal (N-QD-N structure), superconducting (S-QD-N or S-QD-S' structures) or (ferro-) magnetic leads (FM-QD-FM structure) were considered. The effect of Kondo correlations on the conductance of the system with normal, magnetic or superconducting leads has been extensively studied [11–13].

Consider the ultrasmall quantum dot (with very large charging energy and sparse, discrete spectrum), weakly coupled to external electrodes via tunnel barriers. Even for non-magnetic leads, the spin of an electron manifests itself in the appearance of the Kondo effect. If one allows magnetic electrodes, there appear additional interesting effects. The Kondo scale itself depends on the polarisation P of the ferromagnetic leads. The Kondo temperature T_K has been found to be [14]

$$T_K(P) \approx D \exp\left\{ -\frac{1}{N(0)J_0} \frac{\operatorname{arctanh} P}{P} \right\} \tag{1}$$

where $N(0)$ is the total density of states at the Fermi energy, D the effective scale, J_0 the Kondo coupling and $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$ the polarisation.

Quantum-dot-based devices also allow us the study of strongly nonlinear transport. The presence of the signatures of the equilibrium and non-equilibrium Kondo effect in transport through quantum dots has first been predicted theoretically [15, 16] and later confirmed [17] in measurements of the differential conductance $G(V) = dI/dV$. In metals containing magnetic impurities, the Kondo effect manifests itself as an increase of resistance at temperatures $T < T_K$, while in transport through quantum dots one gets enhancement of zero bias conductance $G(0)$ which eventually reaches the unitary limit $2e^2/h$.

The nanostructure consisting of a quantum dot and external electrodes (normal or otherwise) can be modelled by the Anderson Hamiltonian

$$H = \sum_{\mathbf{k},\beta,\sigma} \xi_{\mathbf{k},\beta} c^+_{\mathbf{k},\beta\sigma} c_{\mathbf{k},\beta\sigma} + \sum_\sigma \varepsilon_d d^+_\sigma d_\sigma + U n_{d\uparrow} n_{d\downarrow} + \sum_{\mathbf{k},,\beta,\sigma} (V_{\mathbf{k},\beta} c_{\mathbf{k},\beta\sigma} d^+_\sigma + V^*_{\mathbf{k},\beta} d_\sigma c^+_{\mathbf{k},\beta\sigma}) \tag{2}$$

Here the operators $c_{k\beta\sigma}, c^+_{k\beta\sigma}$ correspond to annihilation and creation of the conduction electrons in the (normal) leads, the energies $\xi_{k\beta\sigma} - \mu_\beta$ in the left ($\beta = L$) or right h.s. ($\beta = R$) electrodes are measured with respect to the chemical potentials μ_L and μ_R. Operators d_σ, d^+_σ refer to the localised electrons on the dot which is characterised by a single energy level ε_d and the charging energy U. The last term in Eq. (2) describes hybridisation between the electrons on the dot and external leads.

A proper technique to study the non-equilibrium transport is the Keldysh non-equilibrium Green's function method [18] and non-crossing approximation to treat many body interactions.

In Figures (1) and (2) we show the temperature dependence of the conductance G and thermopower S of the quantum dot connected to non-magnetic leads. The Green functions have been calculated in the non-crossing approximation and the limit of infinitely large charging energy $U = \infty$ has been assumed. Note the increase of G at low temperatures and its saturation at $T = 0$, where $G = 2e^2/h$ – the unitary limit. The important point is that S changes the sign at temperatures close to the Kondo temperature. It is the appearance of the Kondo resonance which changes the slope of the spectrum of electrons at the Fermi level and this leads to the change of sign of the thermopower.

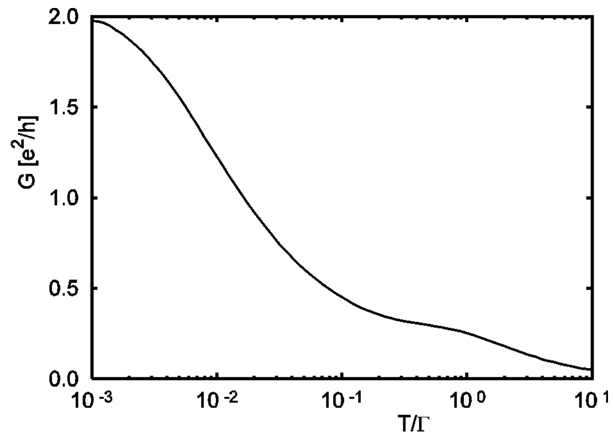

Fig. 1. Temperature dependence of the conductance G of a quantum dot calculated in the non-crossing approximation. Note the increase of G at low temperatures and its saturation at $T = 0$, where $G = 2e^2/h$ – the unitary limit

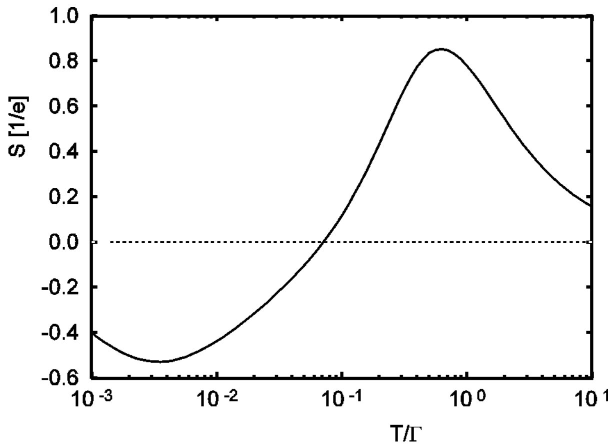

Fig. 2. Temperature dependence of the thermopower S of a quantum dot calculated in the non-crossing approximation. The change of sign of S marks the appearance of the Kondo effect

To understand the change of sign of the thermopower with decreasing temperature one has to note that the Kondo resonance forms slightly above the Fermi energy. It changes the slope of the density of states. The thermopower which depends on the asymmetry (slope) of the density of states at the Fermi level thus changes sign.

Recently, we studied the thermoelectric phenomena of a quantum dot attached to two magnetically polarised [19], as well as nonmagnetic [20] leads. The Kondo effect becomes less pronounced with increasing magnetic polarisation of the leads and this influences all the transport coefficients. It is an interesting observation that the Fermi liquid behaviour is recovered at temperatures $T < T_K$ both for normal as well as magnetic electrodes.

3. The spin Hall effect

The driving force behind the studies of spintronics is the desire to use the electron spin in information storage and processing devices. Spin control, injection and detection require clever tricks. Methods enabling fast generation and manipulation of spin currents by applying electric field are of particular interest. It is the spin Hall effect which has become of great interest. The effect has been analysed in a number of theoretical papers.

The spin Hall effect (SHE) is closely related to the well-known anomalous Hall effect (AHE), frequently observed in ferromagnetic materials. In ferromagnets, the anomalous Hall effect is connected *inter alia* with spin-orbit interaction. The spin up (down) carriers moving under the influence of external voltage in the presence of spin -orbit interaction preferably scatter to the left (right) with respect to original trajectory causing the transverse spin current. This is the essence of the spin Hall effect. Such scattering does not produce an extra charge imbalance and vice versa the spin current, i.e. the movement of spin up electrons in the direction opposite to spin down electrons under the influence of spin-orbit scattering induces an extra perpendicular voltage, which contributes to the anomalous Hall effect.

The SHE has been theoretically predicted in the early seventies [21] and rediscovered a few years ago [22]. Since then, in view of possible applications in spintronic devices it has been very intensively studied theoretically and experimentally [23]. The effects are very subtle. One distinguishes the intrinsic and extrinsic SHE. For the purpose of this paper, it is enough to say that the extrinsic effect requires spin dependent scattering on impurities, while the intrinsic SHE is due to spin-orbit coupling (SOC) terms in single particle Hamiltonian of a clean system. These can be of two types, commonly known as Dresselhaus and Rashba couplings. The Dresselhaus SOC results from the bulk band structure, and is connected with the absence of bulk inversion symmetry (the lattice without an inversion centre) and depends on the material studied. On the other hand, the Rashba SOC develops because of structure inversion asymmetry (no centre of inversion in nanostructure confining potential) and may thus be controlled by electric field applied to, e.g., quantum wells. It has been found that in narrow 2D quantum well of GaSb, the Dresselhaus spin-orbit coupling takes on the form:

$$H_D = \alpha_D(\sigma_x \hat{k}_x - \sigma_y \hat{k}_y) \tag{3}$$

where σ_x, σ_y are the Pauli matrices, $\hat{k}_i$ is the unit vector in the direction i, α_D – the coupling constant (2–20)$\times 10^{-10}$ eV·cm) and diminishes with increasing the width of the well. The Rashba term has a similar structure

$$H_R = \alpha_R(\sigma_x \hat{k}_y - \sigma_y \hat{k}_x) \tag{4}$$

The coupling constant α_R in InAs based quantum well can be as large as (1–6) $\times 10^{-9}$ eV·cm. It is important to realise that in semiconductors the effective spin-orbit coupling may exceed that calculated for an electron in vacuum by 6 orders of magnitude.

In narrow samples, the spin Hall effect leads to spin accumulation at the opposite edges of the sample. Recently, the effect has been observed experimentally *via* optical techniques [24] and more recently by direct measurement [25]. The comparison of experiment with theory, however, is not satisfactory and calls for better understanding of the effect.

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