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Electronic transport in a ferromagnetic single-electron transistor with non-collinear magnetizations in the co-tunnelling regime

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Spin-dependent electronic transport in a ferromagnetic single-electron transistor (FM SET) is studied theoretically in the Coulomb blockade regime [1]. Two external electrodes and the central part (island) of the device are assumed to be ferromagnetic, with the corresponding magnetizations being non-collinear in a general case. First order (sequential) transport is suppressed in the Coulomb blockade regime, so the second order (co-tunnelling) processes give the dominant contribution to the current. The co-tunnelling processes take place *via* four intermediate (virtual) states of the island: two of them are with one extra electron on the central electrode of the device (in the spin-majority or spin-minority subbands), whereas the other two virtual states are with a hole (in the spin-majority or spin-minority subbands) in the central electrode. The co-tunnelling processes create electron-hole excitations of the central electrode, and in a general case they also can create spin excitations. However, we assume relatively fast spin relaxation in the island, hence the spin accumulation is neglected. Basic transport characteristics, like tunnelling current and tunnel magnetoresistance are calculated for an arbitrary magnetic configuration of the system.

Key words: *ferromagnetic single-electron transistor; spin-polarized transport; co-tunnelling*

1. Introduction

Electronic transport through magnetic nanometer-size devices has been extensively studied due to expected future application. In such nanoscale systems, one can manipulate not only a single electron charge, but also a single electron spin. In real double-barrier tunnel junctions, magnetic moments of the electrodes can form a non-collinear magnetic configuration and can have strong influence on the transport characteristics. In our recent paper [2], we have shown that transport characteristics in the sequential tun-

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nelling regime strongly depend on the magnetic configuration of ferromagnetic single-electron transistor (FM SETs).

In this paper, we present the results of our theoretical analysis of spin-dependent co-tunnelling transport in a FM SET whose all three electrodes are ferromagnetic and made of the same material. The corresponding magnetizations are generally non-collinear and oriented in a common plane. In a general case, an external gate voltage can be applied to the island. The gate, however, is neglected in this paper, where we analyze the dependence of electric current and tunnel magnetoresistance (TMR) on the transport voltage and on the angles between magnetizations. Since the analysis is restricted to co-tunnelling processes only, the results are applicable for bias voltage significantly below the resonance (threshold voltage, at which the first Coulomb step appears).

The numerical analysis of the transport characteristics is restricted to the zero temperature limit, with the corresponding transition rates determined from the Fermi golden rule for the second-order transitions. It is also assumed that spin relaxation on the island is sufficiently fast to neglect spin accumulation. The electric current flowing through the device and the resulting tunnel magnetoresistance are calculated for different magnetic configurations of the device.

2. Model and theoretical description

The considered FM SET consists of three electrodes made of the same ferromagnetic material – a small metallic central electrode (called an island) connected by tunnel barriers to two external ferromagnetic electrodes, to which a transport voltage V is applied. Magnetic moments of the electrodes are oriented arbitrary within a common plane.

In our considerations, we take into account only co-tunnelling processes at zero temperature. If the barrier resistances exceed significantly the quantum resistance, $R_j >> R_q = h/e^2$ ($j = 1, 2$), the sequential tunnelling in the Coulomb blockade regime for $T \to 0$ K is exponentially suppressed ($I \propto \exp(\Delta E/kT)$, with ΔE being the increase in the energy), and the dominant contribution to current is due to co-tunnelling processes. The co-tunnelling processes go *via* intermediate (virtual) states of the island. In the case considered, we have four virtual states of the island; two of them are with one extra electron (spin-majority or spin-minority) on the central electrode of the device, whereas the other two virtual states are with a hole (in the spin-majority or spin-minority subbands) in the central electrode. An important property of the co-tunnelling is that the electrons involved in the co-tunnelling processes not only transfer charge, but also create electron-hole excitations of the central electrode (inelastic co-tunnelling). The calculations in this paper are carried out in the limit of fast spin relaxation processes (no spin accumulation on the island). Apart from this, the island is assumed to be large enough to neglect the quantization effects in the island (the relevant energy spectrum can be treated as a continuous one).

The second-order electron tunnelling rate from the spin majority/minority (+/–) subband of the left (L) electrode to the spin majority/minority (+/–) electron channel of the right (R) electrode is given by [3]:

$$\Gamma_{L\to R}^{\pm\to\pm} = \frac{2\pi}{\hbar}\sum_{i,f}\left|\sum_{v}\frac{\langle i|H_T|v\rangle\langle v|H_T|f\rangle}{\varepsilon_v-\varepsilon_i}\right|^2\delta\left(\varepsilon_i-\varepsilon_f\right) \tag{1}$$

where ε_i and ε_f are the energies of initial $|i\rangle$ and final $|f\rangle$ states of the system, ε_v is the energy of the virtual state $|v\rangle$, and $H_T|\rangle$ is the tunnelling Hamiltonian.

Taking into account the local quantization axes in all three electrodes (two external and the central one), and assuming constant (independent of energy) density of states in the leads and constant transfer matrix elements, one can write the tunnelling rate from the spin majority/minority (+/–) subband of the left electrode to the spin majority/minority (+/–) electron channel of the right electrode in the form,

$$\Gamma_{L\to R}^{+\to+} = \frac{\cos^2\frac{\beta}{2}\cos^2\frac{\alpha}{2}}{R_{1,+}^{P}R_{2,+}^{P}}\gamma + \frac{\sin^2\frac{\beta}{2}\sin^2\frac{\alpha}{2}}{R_{1,+}^{AP}R_{2,+}^{AP}}\gamma \tag{2a}$$

$$\Gamma_{L\to R}^{+\to-} = \frac{\cos^2\frac{\beta}{2}\sin^2\frac{\alpha}{2}}{R_{1,+}^{P}R_{2,-}^{AP}}\gamma + \frac{\sin^2\frac{\beta}{2}\cos^2\frac{\alpha}{2}}{R_{1,+}^{AP}R_{2,-}^{P}}\gamma \tag{2b}$$

$$\Gamma_{L\to R}^{-\to+} = \frac{\sin^2\frac{\beta}{2}\cos^2\frac{\alpha}{2}}{R_{1,-}^{AP}R_{2,+}^{P}}\gamma + \frac{\cos^2\frac{\beta}{2}\sin^2\frac{\alpha}{2}}{R_{1,-}^{P}R_{2,+}^{AP}}\gamma \tag{2c}$$

$$\Gamma_{L\to R}^{-\to-} = \frac{\sin^2\frac{\beta}{2}\sin^2\frac{\alpha}{2}}{R_{1,-}^{AP}R_{2,-}^{AP}}\gamma + \frac{\cos^2\frac{\beta}{2}\cos^2\frac{\alpha}{2}}{R_{1,-}^{P}R_{2,-}^{P}}\gamma \tag{2d}$$

where β (α) are the angles between magnetic moments of the left (right) electrode and the island, $R_{1,\pm}^{P}$ ($R_{2,\pm}^{P}$) denotes the spin-dependent resistance of the left (right) barrier in the parallel magnetic configuration, and $R_{1,\pm}^{AP}$ ($R_{2,\pm}^{AP}$) have similar meaning for the antiparallel configuration.

In Eqs. (2a)–(2d) the parameter γ is defined as

$$\gamma = \frac{\hbar}{2\pi e^4}\int d\varepsilon_1 d\varepsilon_2 d\varepsilon_3 d\varepsilon_4 f\left(\varepsilon_1\right)\left[1-f\left(\varepsilon_2\right)\right]f\left(\varepsilon_3\right)\left[1-f\left(\varepsilon_4\right)\right]$$
$$\times\left(\frac{1}{\varepsilon_2-\varepsilon_1+E_1}+\frac{1}{\varepsilon_4-\varepsilon_3+E_2}\right)^2\delta\left(eV+\varepsilon_1-\varepsilon_2+\varepsilon_3-\varepsilon_4\right) \tag{3}$$

where $f(\varepsilon)$ is the Fermi distribution function, ε_1 and ε_4 are the energies measured from the Fermi level of the left and right electrodes, whereas ε_2 and ε_3 are measured from the Fermi level of the island. Apart from this, E_1 and E_2 are the changes in the electrostatic energies associated with the two different virtual states:

$$E_1 = e\left(\frac{\frac{e}{2} - VC_2}{C_\Sigma}\right) \quad \text{and} \quad E_2 = e\left(\frac{\frac{e}{2} - VC_1}{C_\Sigma}\right)$$

where C_1 and C_2 are the capacitances of the left and right junctions, and $C_\Sigma = C_1 + C_2$. In the zero temperature approximation and for small voltages ($eV << E_1$, E_2), the above integral can be calculated analytically and one gets

$$\gamma \approx \frac{\hbar}{12\pi e}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)^2 V^3 \tag{4}$$

In the co-tunnelling regime, the electric current flowing through the system can be then calculated as

$$I_{L\to R}(V) = e\sum_{\sigma=+,-}\sum_{\sigma'=+,-}\left[\Gamma_{L\to R}^{\sigma'\to\sigma} - \Gamma_{R\to L}^{\sigma\to\sigma'}\right] \tag{5}$$

where $\Gamma_{R\to L}^{\sigma\to\sigma'}$ is the spin-dependent tunnelling rate for backward processes.

Let us now present some numerical results on electric current and tunnel magnetoresistance, obtained with the formulas derived above.

3. Numerical results and discussion

Using Equation (5), we can calculate co-tunnelling current for any magnetic configuration of the device, and hence also the tunnel magnetoresistance effect defined by the ratio [4]

$$TMR = \frac{I(\beta = 0, \alpha = 0)}{I(\beta, \alpha)} - 1 \tag{6}$$

where $I(\beta,\alpha)$ is the current flowing in the non-collinear magnetic configuration (in the parallel configuration we have $\beta = 0$ and $\alpha = 0$).

In Figure 1, we show the results of numerical calculations of electric current flowing through the system as a function of the bias voltage for selected values of the angles. The upper part corresponds to $\beta = 0$, whereas the lower one to $\beta = \alpha$. The electric current in the Coulomb blockade regime varies as the third power of the voltage, $I \propto V^3$.

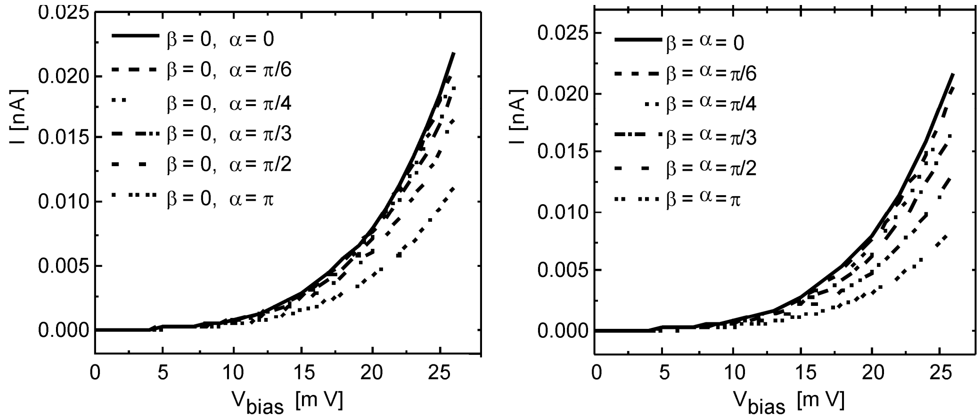

Fig. 1. The electric current as a function of the bias voltage in a FM SET with non-collinear magnetizations for different angles between magnetic moments. The parameters taken in numerical calculations are: $C_1 = C_2 = 1$ aF, $R_1^{p,+} = 0.5\ \mathrm{M\Omega}$, $R_1^{p,-} = 0.1\ \mathrm{M\Omega}$, $R_2^{p,+} = 25\ \mathrm{M\Omega}$, $R_2^{p,-} = 5\ \mathrm{M\Omega}$, whereas $R_i^{ap} = \sqrt{R_i^{p,+} R_i^{p,-}}$ for $i = 1, 2$. The bias voltage was applied symmetrically: $V_2 = -V_1 = -V/2$.

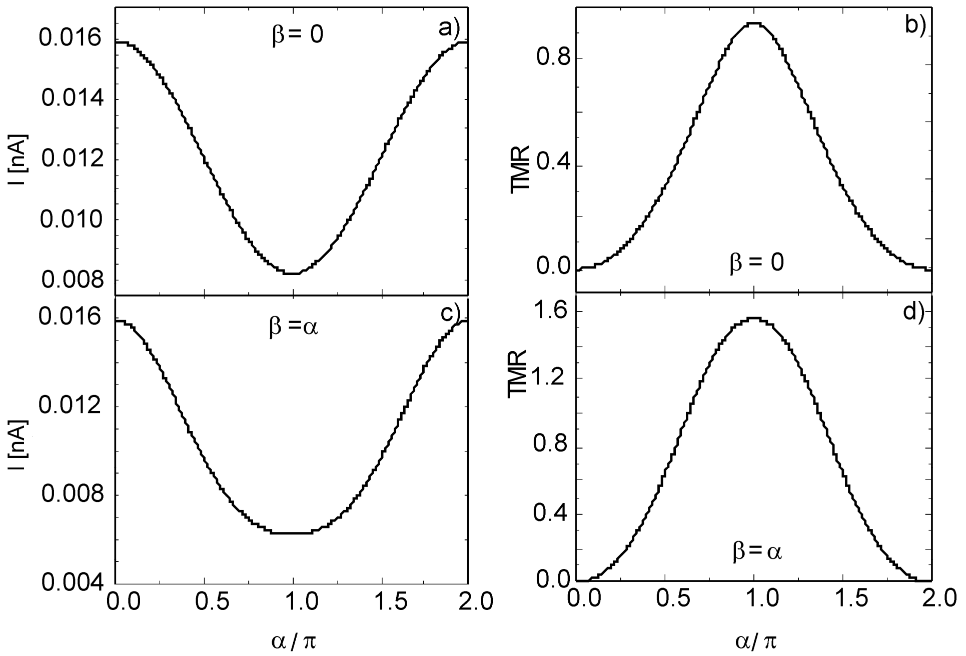

Fig. 2. The dependences of electric current (a), (c) and TMR (b), (d) on the angles between magnetizations, calculated for the bias voltage $V = 24$ mV. The other parameters are the same as in Fig.1.

The currents flowing in the system depend on its magnetic configuration, and this difference gives rise to a nonzero TMR which, however, is constant over the whole bias range (this may change in systems where the assumptions made in this paper are not obeyed). The angular dependence of current and the associated TMR is shown in Fig. 2 which clearly shows that electric current has a minimum in the antiparallel magnetic configuration, which corresponds to a maximum in TMR.

4. Conclusions

In this paper, we have studied transport characteristics of a single-electron transistor in the co-tunnelling regime. The analyzed device consists of a ferromagnetic particle (island) and two ferromagnetic external electrodes (made of the same material), whose magnetizations are oriented arbitrary. We have calculated electric current flowing through such a device and the corresponding TMR. Bias dependence of electric current reveals its variation with the bias as $I \propto V^3$, while the TMR effect is independent of the bias voltage. Furthermore, the current flowing through the system, as well as the TMR, strongly depend on the angles between magnetizations.

References

[1] GRABERT H., DEVORET M.H., *Single Charge Tunnelling*, NATO ASI Series B, Vol. 294, Plenum Press, New York, 1992.
[2] WIŚNIEWSKA J., WEYMANN I., BARNAŚ J., Mater. Sci.-Poland, 22 (2004), 461.
[3] FEYMANN R.P., HIBBS A.R., *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, 1965.
[4] JULLIERE M., Phys. Lett. A, 54 (1975), 225.

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