

# Phase diagrams and properties of the ground state of the anisotropic Kondo lattice model

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The properties of the half-filled Kondo lattice model with anisotropic intersubsystem exchange interaction  $J_{XY}$  and  $J_Z$  are discussed. The phase diagram and ground state characteristics of the system are determined within a variational mean-field approximation for the case of rectangular density of states (DOS) for itinerant electrons. The ground state phase diagram is found to exhibit five different phases: Kondo singlet state, two planar antiferromagnetic phases, and two Ising antiferromagnetic (Néel) phases.

Key words: *Kondo lattice; exchange interaction; phase diagram*

## 1. Introduction

The Kondo lattice model (KLM) is one of the most common models for heavy fermion materials, Kondo insulators and also for manganites (in its ferromagnetic version). In the model, charge fluctuations for localized electrons are suppressed, which leads to a coupled electron–spin system. Crucial for physics of this system is a strong competition between demagnetization resulting from the Kondo effect and magnetism, which tends to yield magnetic orderings. Up to now, most of the studies of this model focused on the case of isotropic exchange interaction [1–6].

In this paper, we present some of our results concerning the case of the Kondo lattice model (KLM) with anisotropic exchange couplings  $J_{XY}$  and  $J_Z$ . The model is defined by the following Hamiltonian:

$$H = \sum_{\langle ij \rangle \sigma} t_{ij} c_{i\sigma}^+ c_{j\sigma} + \sum_i J_Z S_i^z \sigma_i^z + \frac{1}{2} J_{XY} (S_i^+ \sigma_i^- + S_i^- \sigma_i^+) \quad (1)$$

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where  $S_i^\alpha$  and  $\sigma_i^\alpha$  are the spin operators for localized  $d$  electrons at  $i$ th site and those for itinerant  $c$  electrons, respectively:

$$S_i^\mu = \frac{1}{2} \sum_{\sigma\sigma'} d_{i\sigma}^\dagger \tau_{\sigma\sigma'}^\mu d_{i\sigma'}, \quad \sigma_i^\mu = \frac{1}{2} \sum_{\sigma\sigma'} c_{i\sigma}^\dagger \tau_{\sigma\sigma'}^\mu c_{i\sigma'}$$

with the Pauli matrices  $\tau_{\sigma\sigma'}^\mu$ , and with a local constraint  $d_{i\uparrow}^\dagger d_{i\uparrow} + d_{i\downarrow}^\dagger d_{i\downarrow} = 1$ , such that every  $d$  orbital is always occupied by just one electron. The bandwidth parameter  $D$  of  $c$  electron band is defined by  $2D = 2zt$ , where  $z$  is the number of nearest neighbours (nn).

We performed a detailed analysis of the phase diagrams and thermodynamic properties of model (1) for  $d$ -dimensional hypercubic lattices and arbitrary, positive and negative  $J_{XY}$  and  $J_Z$  [7]. In the analysis, we used an extended mean-field approximation (MFA-HFA), analogous to that used in the treatment of the isotropic Kondo lattice model [2, 4]. Below, we only quote the main results of this investigation, concentrating on the case of a half-filled electron band. We restrict our analysis to the pure phases and assume rectangular density of states (DOS) for  $c$  electron band.

Considered phases are characterised by the following order parameters:

- planar antiferromagnetic (AF<sub>XY</sub>)

$$\sigma_Q^{XY} \neq 0, \quad S_Q^{XY} \neq 0$$

where

$$\sigma_Q^{XY} = \frac{1}{2N} \sum_i \langle \sigma_{ic}^+ \rangle e^{-iQR_i}, \quad S_Q^{XY} = \frac{1}{2N} \sum_i \langle S_{id}^+ \rangle e^{-iQR_i}$$

$$\sigma_{ic}^+ = c_{i\uparrow}^\dagger c_{i\downarrow}, \quad S_{id}^+ = d_{i\uparrow}^\dagger d_{i\downarrow}, \quad \vec{Q} = \left( \frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a} \right)$$

- uniaxial AF state (AF<sub>Z</sub>)

$$\sigma_Q^z \neq 0, \quad S_Q^z \neq 0$$

where

$$\sigma_Q^z = \frac{1}{N} \sum_i \langle 2\sigma_{ic}^z \rangle e^{-iQR_i}, \quad S_Q^z = \frac{1}{N} \sum_i \langle 2S_{id}^z \rangle e^{-iQR_i}$$

- Kondo state (K)

$$\lambda = \frac{1}{2N} \sum_i (\langle d_{i\sigma}^\dagger c_{i\sigma} \rangle + hc) = \frac{1}{2N} \sum_k (\langle d_{k\sigma}^\dagger c_{k\sigma} \rangle + hc)$$

## 2. Results and discussion

The ground state phase diagram of the considered model calculated for rectangular density of states (DOS) is shown in Fig. 1. In the next two figures, the plots of the order parameters and the quasi-particle gaps in the excitation spectrum as a function of increasing interactions are presented: in Fig. 2 as a function of  $J_{XY}/2D$  for  $J_Z = 0$ , whereas Fig. 3 as a function of  $J_Z/2D$  for  $J_{XY}/2D = 0.14$ .

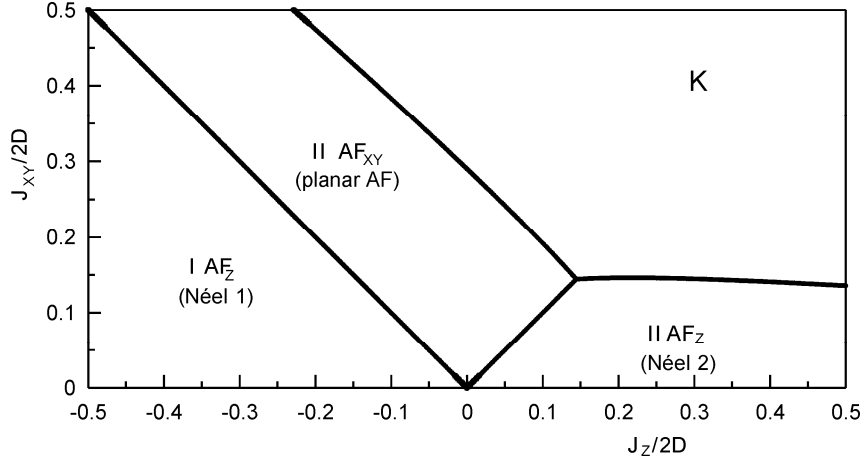


Fig. 1. Ground state phase diagram of the anisotropic Kondo lattice model at half-filling, plotted as a function of  $J_{XY}/2D$  and  $J_Z/2D$ , for rectangular DOS

The diagram in Fig. 1 has been derived for  $J_{XY} \geq 0$  and consists of four phases: K Kondo singlet state, I  $AF_Z$  Néel 1 (Ising AF with parallel sublattice uniaxial magnetizations of spins and electrons), II  $AF_Z$  Néel 2 (Ising AF with antiparallel sublattice magnetizations) and II  $AF_{XY}$  (planar AF with antiparallel sublattice XY magnetization).

For  $J_{XY} \leq 0$  the diagram has the same form with the replacement of II  $AF_{XY}$  by I  $AF_{XY}$  (planar antiferromagnet with parallel sublattice XY magnetizations of spins and electrons).

Let us shortly point out the main conclusions:

- For small values of  $J_{XY}/2D$  and  $J_Z/2D$  the ground states are  $AF_{XY}$ , if  $|J_{XY}| > |J_Z|$  and  $AF_Z$  if  $|J_{XY}| < |J_Z|$ , for both signs of  $J_{XY}$  and  $J_Z$ .
- For  $|J_{XY}| > |J_Z|$  with increasing  $|J_{XY}|/2D$  the system exhibits a transition from  $AF_{XY}$  to Kondo state at  $(|J_{XY}|/2D)_c$ . The value  $(|J_{XY}|/2D)_c$  depends on the strength of  $J_Z$ :  $J_Z > 0$  reduces this critical value, whereas  $J_Z < 0$  enhances it (cf. Fig. 1).
- For  $0 < |J_{XY}| < |J_Z|$ , the system remains in the I  $AF_Z$  state for any  $|J_Z|/2D$ , if  $J_Z < 0$ , whereas for  $J_Z > 0$  the increase of  $|J_Z|/2D$  yields a transition  $II AF_Z \rightarrow K$ , at the critical value  $(|J_{XY}|/2D)_c$  which slowly decreases with increasing  $J_Z/2D$  (cf. Fig. 1).

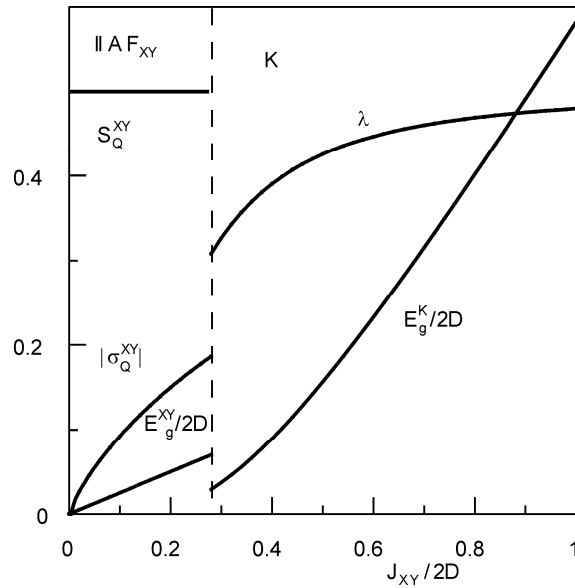


Fig. 2. Variation of the  $\parallel \text{AF}_{XY}$  and Kondo (K) order parameters and the quasiparticle gap at  $T = 0$  as a function of  $J_{XY}/2D$  for  $J_Z = 0$

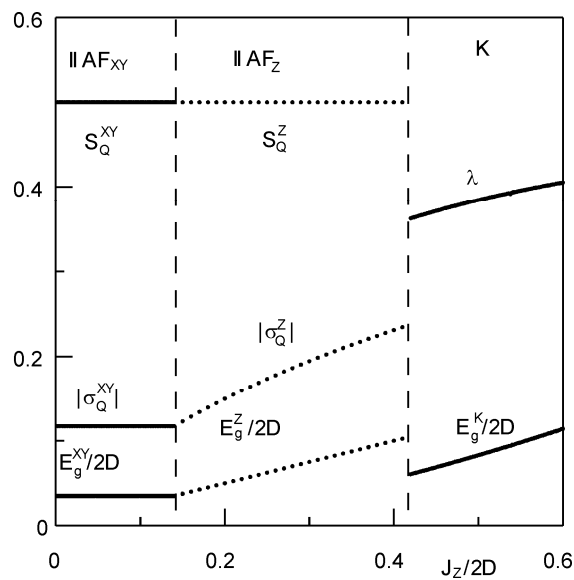


Fig. 3. Variation of the  $\parallel \text{AF}_{XY}$ ,  $\parallel \text{AF}_Z$  and Kondo (K) order parameters and the quasiparticle gap at  $T = 0$  as a function of  $J_Z/2D$  for  $J_{XY}/2D = 0.14$

- In the considered case of half filled electron band at  $T = 0$  all phases are nonmetallic, with the quasi-particle gaps:  $E_g = |J_Z S_Q^z|$ , for  $\text{AF}_Z$  states,  $E_g = |J_{XY} S_Q^{XY}|$ , for

$AF_{XY}$  states and  $E_g = \sqrt{D^2 + X^2} - D$ , where  $X = (2J_{XY} + J_Z)\lambda$ , for K state. Notice a sharp decrease of the gap at the transitions from  $AF_{XY}$  and  $AF_Z$  to Kondo state (cf. Figs. 2 and 3).

• With increasing  $J_Z/2D$  ( $-\infty < J_Z/2D < \infty$ ) for fixed  $J_{XY}/2D > 0$  one can observe the following sequences of transitions (cf. Figs. 1, 3):

$$I AF_Z \rightarrow AF_{XY} \rightarrow II AF_Z \rightarrow K, \quad \text{if } |J_{XY}/2D| < (J_{XY}/2D)_0$$

$$I AF_Z \rightarrow AF_{XY} \rightarrow K, \quad \text{if } |J_{XY}/2D| > (J_{XY}/2D)_0,$$

where  $(J_{XY}/2D)_0 \approx 0.145$  for a rectangular DOS.

Our present studies of model (1) concentrate on the effects of external magnetic field and take into consideration mixed ordered phases.

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