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Theoretical analysis of transport in ferromagnetic single -electron transistors in the sequential tunnelling regime*

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Spin-dependent transport properties of a single-electron transistor have been analysed, whose two external electrodes and the central part (island) are ferromagnetic. Based on the real-time diagrammatic technique, all transport contributions have been calculated up to the first order in the coupling strength between the island and the leads – this comprises a sequential tunnelling. Relevant occupation probabilities of different charge states are determined from the generalized master equation in the Liouville space. Assuming that spin relaxation processes on the island are sufficiently fast to neglect spin accumulation, we analyze electric current flowing through the system in the parallel and antiparallel magnetic configurations, as well as the resulting tunnel magnetoresistance. We show that transport characteristics of ferromagnetic single-electron transistors exhibit a strong dependence on the magnetic configuration of the system. Furthermore, we also demonstrate that the bias dependence of both the differential conductance and tunnel magnetoresistance displays an oscillatory-like behaviour resulting from single-electron charging effects.

Key words: *ferromagnetic single-electron transistor; spin-dependent transport; real-time diagrammatic technique*

1. Introduction

We consider spin-dependent transport properties of a ferromagnetic single-electron transistor (FM SET) whose two external electrodes as well as the central electrode (island) are ferromagnetic. Based on the real-time diagrammatic technique [1–3], we analyze spin-dependent transport in the sequential tunnelling regime. In this transport regime, if the applied bias voltage is above a certain threshold voltage, the current flows due to consecutive, uncorrelated single-electron tunnelling events. On the other hand, if the applied transport voltage is below the threshold voltage, the current is exponentially suppressed due to the Coulomb interaction which prevents further tunnelling and gives rise to the Coulomb blockade effect. Apart from the Coulomb block-

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ade phenomena, charging with single electrons leads to the so-called Coulomb staircase characteristics of the current versus the bias voltage as well as the sawtooth-like dependence of the current on the gate voltage Coulomb oscillations [4]. In addition, if the electrodes are ferromagnetic, one finds further interesting effects, resulting from the interplay of the single-electron charging and spin-dependence of tunnelling processes such as oscillations of tunnel magnetoresistance (TMR), spin accumulation, etc. [5–8].

Assuming that the spin relaxation processes on the island are sufficiently fast to neglect the effects of spin accumulation, we have analyzed the bias and gate voltage dependence of the electric current and differential conductance of FM SET in the parallel and antiparallel magnetic configurations as well as the resulting TMR effect. We show that the TMR exhibits an oscillatory-like behaviour on the bias voltage, while it changes periodically with the gate voltage.

In Section 2, we define the model and present its theoretical description. Section 3 contains general results calculated with the aid of the real-time diagrammatic technique and discussion. Summary is given in Section 4.

2. Model and theoretical description

A scheme of a ferromagnetic single-electron transistor considered in the paper is shown in Fig. 1. It consists of a mesoscopic metallic island coupled by tunnelling junctions to two external electrodes, as well as to an external nonmagnetic gate voltage. Both the island and the leads are assumed to be built of ferromagnetic materials. It is also assumed that the system can be in two different magnetic configurations: the parallel and antiparallel ones. In the parallel configuration, all the magnetizations are aligned, whereas in the antiparallel configuration the magnetization of the island is antiparallel to magnetic moments of the leads (Fig. 1).

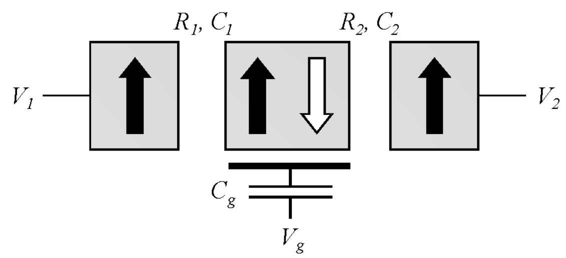

Fig. 1. A scheme of the ferromagnetic single-electron transistor consisting of two ferromagnetic external electrodes and a ferromagnetic island between them, while the gate is nonmagnetic. It is assumed that the system can be either in parallel or antiparallel magnetic configuration. The parameters are as indicated in the figure

The tunnelling of electrons through small metallic island in ferromagnetic single -electron transistor reveals, associated with low capacitance of the junctions, a strong dependence on the charging energy of the system given by [4]

$$E_{ch}(N,V) = \frac{\left(Ne - C_1V_1 - C_2V_2 - C_gV_g\right)^2}{2C} \quad (1)$$

where e is the electron charge, N denotes the number of excess electrons occupying the island, $C = C_1 + C_2 + C_g$ is the total capacitance of the island, C_1 and C_2 are the capacitances of the first and second junctions, while C_g denotes the gate capacitance. When the charging energy is larger than the thermal energy, transport characteristics reveal the effects associated with discreteness of charge, such as Coulomb blockade, Coulomb oscillations, etc. [4]. Furthermore, if the transparency of tunnel barriers is low and the resistance of a single barrier is much higher than the quantum resistance, $R_Q = h/e^2$, transport through the system is dominated by incoherent, sequential tunnelling processes, when the electrons are transferred through the system one by one. This transport regime can be described in the lowest-order perturbation with respect to the coupling strength of the island to the leads. Moreover, we also note that this transport regime is relevant to many current experiments [9]. In this analysis we assume that spin relaxation processes on the island are sufficiently fast to neglect spin accumulation on the island. We also assume that the system is symmetrically biased, $V_1 = -V_2 = V/2$.

In order to determine the transport characteristics, we employ a diagrammatic technique in a real time [1–3] to identify processes of sequential tunnelling in the FM SET. This technique consists in a systematic perturbation expansion of the density matrix elements and the current operator with respect to the coupling between the island and the leads. The transport properties of the system are governed by the time evolution of the density matrix, which is usually visualized graphically as a sequence of irreducible diagrams on the Keldysh contour [1–3]. Each diagram has a certain number of tunnelling lines, which indicate respective tunnelling processes between the island and the leads. Since we consider only the first-order processes, the diagrams in the sequential tunnelling approximation have only one tunnelling line, corresponding to a sequential tunnelling event. Each tunnelling line gives the golden-rule rate

$$\alpha_r^{\pm,\sigma}\left(\omega\right) = \pm\alpha_0^{r,\sigma}\frac{\omega-\mu_r}{e^{\pm\beta(\omega-\mu_r)}-1}, \quad (2)$$

where

$$\alpha_0^{r,\sigma} = \frac{h}{4\pi^2e^2}\frac{1}{R_{r,\sigma}} = \frac{R_Q}{4\pi^2R_{r,\sigma}}$$

$R_{r,\sigma}$ denotes spin-dependent resistance of the $r = 1, 2$ junction, ω is the energy of tunnelling lines in diagrams, μ_r denotes the electrochemical potential of the electrode r, and $\beta = 1/k_BT$. The sign of $\alpha_r^{\pm,\sigma}(\omega)$ depends on the direction of tunnelling line (+ for lines directed backward or – for lines directed forward with respect to the closed time path in diagram).

Because we consider only the spin-conserving tunnelling processes and collinear magnetic configurations of the system, the density matrix of the system is diagonal; its elements, P_χ, correspond then to the probability of finding the system in a certain state χ in charge space. The relevant probabilities in a stationary state can be found by using the following master-like equation [1–3]

$$0 = \sum_{\chi'} P_{\chi'} \Sigma_{\chi'\chi} \tag{3}$$

where the object needed as an input is the Laplace transform of the generalized transition rate $\Sigma_{\chi',\chi}(t',t)$ from a state χ' (initial charge state of a diagram) at time t' to state χ (final charge state of a diagram) at time t. This rate is the sum over all irreducible diagrams [1–3] contributing to the transport through the FM SET and corresponds to the self-energy of the Dyson equation for the full propagator of the system. The value of any first-order diagram is added to the appropriate matrix element of the self-energy matrix $\Sigma_{\chi',\chi}$. The self-energies can be defined as

$$\Sigma_{\chi',\chi}(t',t) = \sum_r \left\{ \Sigma^{r+}_{\chi',\chi}(t',t) + \Sigma^{r-}_{\chi',\chi}(t',t) \right\} \tag{4}$$

here $\Sigma^{r+}_{\chi',\chi}$ includes all those diagrams which contribute to $\Sigma_{\chi',\chi}$ with the rightmost vertex on the upper propagator and rightmost contraction line describing tunnelling out of electrode through junction r, as well as diagrams with rightmost vertex on the lower propagator and the rightmost process describing tunnelling into lead r; $\Sigma^{r-}_{\chi',\chi}$ is given by summing correspondingly the rest of diagrams. For the case of FM SET and in the sequential tunnelling regime, we get

$$\Sigma_{\chi,\chi} = -\Sigma_{\chi,\chi-1} - \Sigma_{\chi,\chi+1} = -2\pi i \sum_{r,\sigma} \alpha_r^{-,\sigma}\left(E_\chi - E_{\chi-1}\right) - 2\pi i \sum_r \alpha_r^{+,\sigma}\left(E_{\chi+1} - E_\chi\right) \tag{5}$$

Having determined the respective first-order self-energies, one can calculate both the occupation probabilities of the island as well as the sequential current flowing through the system. The stationary current flowing through the barrier r is given by [1–3]

$$I_r = -\frac{ie}{\hbar} \sum_{\chi,\chi'} P_{\chi'} \Sigma^{r+}_{\chi',\chi} = \frac{ie}{\hbar} \sum_{\chi,\chi'} P_{\chi'} \Sigma^{r-}_{\chi',\chi} \tag{6}$$

3. Results and discussion

Using Equation (6) one can calculate the current flowing through the system in the parallel and antiparallel magnetic configurations. Due to the spin dependence of the diagrams, the current flowing in the parallel configuration I_p is generally larger than the current flowing through the system in the antiparallel configuration I_{ap}. This difference gives rise to a nonzero tunnel magnetoresistance (TMR) effect. The TMR is defined as [5, 10]:

$$TMR = \frac{I_p}{I_{ap}} - 1 \tag{7}$$

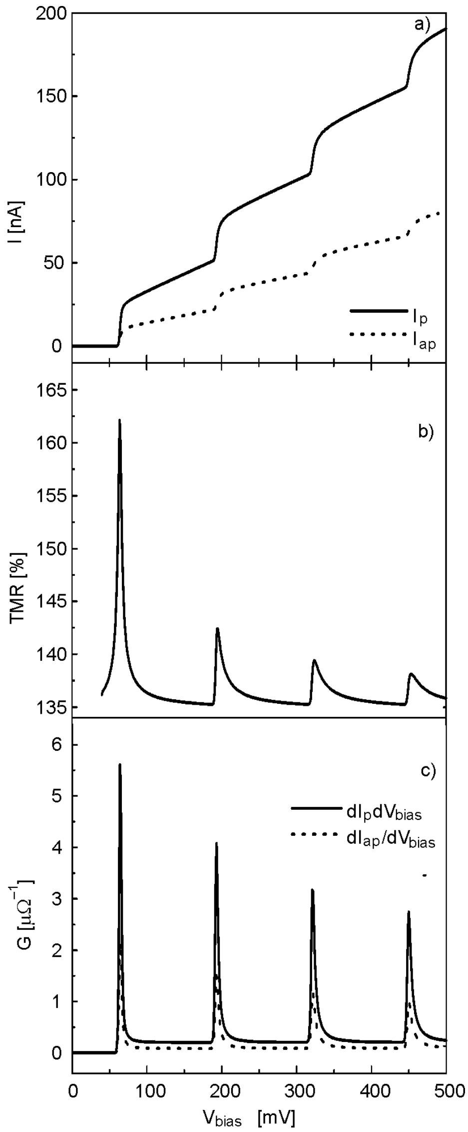

Fig. 2. The bias voltage dependence of electric current (a), differential conductance (c) in the parallel and anti-parallel magnetic configurations, as well as the resulting TMR (b). The parameters assumed in numerical calculations are: $C_1 = C_2 = 1$ aF, $C_g = 0.5$ aF, $T = 4.2$ K and $V_g = 0$, while the barrier resistances: $R_1^{p,+} = 20000R_Q$, $R_1^{p,-} = 100R_Q$, $R_2^{p,+} = 20R_Q$, $R_2^{p,-} = 0.5R_Q$, whereas $R_r^{ap} = \sqrt{R_r^{p,+}R_r^{p,-}}$ for $r = 1.2$

In the following, we present and discuss results of our numerical calculations of transport characteristics of FM SET. In Figure 2a, we show the current flowing through the system as a function of the bias voltage for both parallel and antiparallel magnetic configurations.

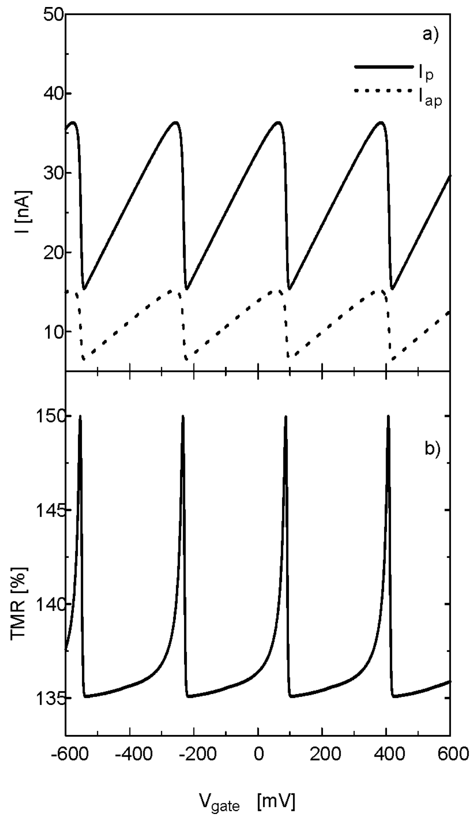

Fig. 3. The electric current (a) and TMR (b) in function of the gate voltage, calculated for the bias voltage $V = 50$ mV. The other parameters are the same as in Fig. 2

First of all, one can see that the sequential current is blocked for small bias voltages – this is the Coulomb blockade effect. When the transport voltage is above the threshold voltage, transport is allowed and electrons can tunnel one by one through the system. By increasing the bias voltage further, additional charge states start participating in transport, which leads to the so-called Coulomb staircase *I–V* characteristic. The occurrence of additional charge states in the energy window provided by transport voltage is also visible in the bias dependence of the differential conductance. Each time a next step in the *I–V* curve appears, there is a peak in differential conductance. This is shown in Fig. 2c. Furthermore, as one can see from Fig. 2a, the current flowing through the system in the parallel configuration is larger than that in the antiparallel one. This results from the fact that in the antiparallel configuration the majority (minority) electrons of the leads tunnel to the minority (majority) electron band of the island, which effectively leads to the suppression of current, as compared to the parallel configuration. This difference gives rise to TMR which is shown in Fig. 2b. The bias dependence of the TMR displays characteristic peaks for voltages corresponding to consecutive steps in the *I–V* curves (Fig. 2a). This leads to an oscillatory-like dependence of the TMR on the bias voltage. Moreover, it is interesting to note that the amplitude of these oscillations decreases with increasing the transport voltage. The oscillations of TMR with the bias voltage are a result of the interplay of spin-dependent tunnelling and single-electron charging effects.

In addition, in Figure 3, we present the gate voltage dependence of the current in the parallel and antiparallel configurations as well as the TMR. By sweeping the gate voltage, one changes the number of excess electrons on the island. Each time additional single electron is allowed to tunnel to the island, there is an enhancement of the current. This leads to the sawtooth-like oscillations of the current. This effect is visible in both magnetic configurations as it does not result from ferromagnetism of electrodes but is associated with single-electron charging phenomena. The gate voltage dependence of the TMR is shown in Fig. 3b. Similarly as the current, the gate voltage dependence of the TMR also displays characteristic oscillations. This effect may be of importance in future magnetoelectronic devices, as be changing the gate voltage one can tune the magnitude of the TMR effect.

4. Conclusions

Theoretical analysis has been carried out of transport properties of the ferromagnetic single-electron transistor consisting of an ferromagnetic island and two ferromagnetic external electrodes. The considerations were based on the real-time diagrammatic technique. By evaluating the contributions coming from different first order diagrams, we determined the current flowing through the FM SET for arbitrary transport and gate voltages. It has been shown that all the transport characteristics, i.e. the current, TMR and differential conductance, strongly depend on the magnetic configuration of the system and reveal the effects associated with the Coulomb blockade, as well as the Coulomb staircase and Coulomb oscillations.

We believe that the present analysis will be a good starting point to extend the considerations to include the higher-order contributions, e.g., due to co-tunnelling processes. This would enable one to study the transport properties of FM SETs also in the Coulomb blockade regime.

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