Materials Science-Poland, Vol. 26, No. 4, 2008

Mesoscopic rings. Multi-states induced by quantum thermal fluctuations

J. DAJKA[*], L. MACHURA, S. ROGOZIŃSKI, J. ŁUCZKA

Institute of Physics, University of Silesia, 40-007 Katowice, Poland

When temperature of a conducting mesoscopic ring is decreased, the magnetic flux states can change drastically, e.g. from bistable to multistable states. Possibility of the application of the multistability of the magnetic flux is emphasised in the information-theoretical context. A proposal of the quantum kinetics description is qualitatively discussed.

Key words: *persistent current; quantum Smoluchowski limit; qutrit*

Mesoscopic multiply connected samples such as conducting rings, cylinders or tori exhibit quantum size phenomena originating from the Aharonov–Bohm effects. One of the most spectacular phenomena is a persistent current [1] which is a direct manifestation of quantum phase coherence of the current carriers over a mesoscopic length scale. At a non-zero temperature, $T > 0$, the phase coherence of carriers is weakened and the samples exhibit the dissipative Ohmic contribution to the total current. There is a regime of temperatures in which both phase-coherent and dissipative Ohmic currents coexist. Such currents generate a magnetic flux. Its properties can be studied via modelling based on kinetic equations like Langevin equations [2]. The long-time, steady states of the magnetic flux are characterized by properties of the stationary probability density obtained from the corresponding Langevin equation. For instance, if the stationary probability density is bimodal, i.e., it has two maxima it reflects the possibility of occurrence of non-zero magnetic fluxes and in consequence the currents. One can notice an obvious analogy to the Josephson-based flux qubits [3]. There are serious advantages for using non-superconducting samples like rings as a building blocks for qubit storage because due to a small size they seem to be more stable with respect to decoherence and dephasing [4]. For higher temperatures, when

[*]Corresponding author, e-mail: dajka@phys.us.edu.pl

the Ohmic current dominates over the persistent current, thermal fluctuations can be modelled as classical white noise with a state-independent intensity determined by the fluctuation–dissipation relation. For moderate and low temperatures, when persistent current starts to dominate over the Ohmic current, this modelling is insufficient and leading quantum corrections to thermal fluctuations should be incorporated. This regime can be described by the so-called quantum Smoluchowski equation [5] in which quantum corrections modify both the diffusion and drift coefficients [6, 7]. The resulting steady states of the system can change drastically, e.g. the bistable states evolve to the tristable ones when the temperature decreases. This opens the door for using non-superconducting rings in qutrit architecture with potential applications, e.g. in the quantum cryptography [8].

The magnetic flux ϕ threadeding a mesoscopic system of the ring topology, operating in the quantum Smoluchowski regime, can be described as a noisy overdamped *RLC* circuit driven by a non-linear "force" created by the persistent current [9]. In this regime, the ϕ dynamics is described by the Langevin equation in the dimensionless form [9, 10]:

$$\frac{dx}{ds} = -\frac{dV_{\text{eff}}(x)}{dx} + \sqrt{2D(x)}\xi(s) \tag{1}$$

where the rescaled flux $x = \phi/\phi_0$ and rescaled time $s = t/\tau_0$. The flux quantum $\phi_0 = h/e$ is the ratio of the Planck constant h and the charge of the electron. The characteristic time $\tau_0 = L/R$, where L is the self-inductance and R is the resistance of the ring, respectively.

The effective "potential" $V_{\text{eff}}(x)$ takes the form

$$V_{\text{eff}}(x) = V(x) + \frac{1}{2}\lambda B''(x), \qquad V(x) = \frac{1}{2}(x - x_e)^2 + B(x) \tag{2}$$

where

$$B(x) = \alpha\sum_{n=1}^{\infty}\frac{A_n(T_0)}{2n\pi}\left(p\cos(2n\pi x) + (1-p)\cos\left(2n\pi\left(x+\frac{1}{2}\right)\right)\right) \tag{3}$$

The dimensionless amplitude of the coherent current $\alpha = LI_0/\phi_0$ with I_0 being the maximum amplitude of the coherent part of the current at zero temperature. The temperature is rescaled with respect to T^* which corresponds to the size induced energy gap at the Fermi surface, i.e. $T_0 = T/T^*$. The detailed form of $A_n(T_0)$ (cf. e.g., [2]) is not reproduced here.

The modified diffusion coefficient $D(x)$ with the Maxwell demon successfully exorcised [6] ensures the equilibrium character of quantum thermal fluctuations:

$$D(x) = \beta^{-1}\left(1 - \lambda\beta\left(1 + B''(x)\right)\right)^{-1} \tag{4}$$

The dimensionless parameters are: $1/\beta = k_BT/2E_m = k_0T_0$, where the elementary magnetic flux energy $E_m = \phi_0^2/2L$ and $k_0 = k_BT^*/2E_m$ is the ratio of two characteristic energies. The δ-correlated stochastic force $\xi(s)$ is zero-mean Gaussian white noise of unit intensity. The prominent parameter [10]

$$\lambda = \lambda_0\left(\gamma + \ln\left(1 + \frac{\varepsilon}{T_0}\right)\right), \quad \lambda_0 = \frac{R}{R_0}, \quad \varepsilon = \frac{E_0}{k_BT^*} \tag{5}$$

characterizes quantum fluctuations in the magnetic flux space. Here, γ is the Euler constant, the resistance quantum $R_0 = \pi\phi_0^2/\hbar$ is expressed by the fundamental constants and the energy $E_0 = \hbar/2\pi CR$ characterizes the CR properties of the ring.

The stationary probability density $P(x)$ can be obtained from the Fokker–Planck equation corresponding to the Langevin equation (Eq. (1)). It takes the form

$$P(x) \propto D^{-1}(x)\exp\left(-\Phi(x)\right) \tag{6}$$

where the generalized thermodynamic potential is

$$\Phi(x) = \int \frac{V'_{\mathrm{eff}}(x)}{D(x)}\,dx \tag{7}$$

The stationary state (6) is a thermal equilibrium state. However, due to both the x dependence of the modified diffusion coefficient $D(x)$ and the temperature dependence of the effective potential $V_{\mathrm{eff}}(x)$, it is not a Gibbs state (which is characterized by the distribution $P_G(x) \propto \exp(-\beta V(x))$.

There are parameter regimes when in the formal absence of the quantum fluctuations, i.e. in the $\lambda \to 0$ limit, the system is bistable reflecting the possibility of occurrence of self-sustaining fluxes. There is one-to-one correspondence between probability density and the exponentiated deterministic potential $V(x)$. Passing into the quantum Smoluchowski limit characterized by a non-zero λ with effective potentials, this correspondence is lost: the steady state at sufficiently low temperatures becomes mulitistable with three central peaks dominating the other ones (Fig. 1).

The peaks in the probability distribution correspond to stationary states of the system. A natural measure of the stability of a formally meta-stable state $x = x_m$ is its lifetime $\tau(x_m)$ calculated as an expected time of the "particle" starting at $x = x_m$ to leave the well of the effective potential [10]. The plot presenting numerical results is shown in Fig. 2. The lifetime of two fundamental peaks at $x_m = 0, \pm 1/2$ exceeds the lifetime of $x_m = 1$ by almost one order of magnitude.

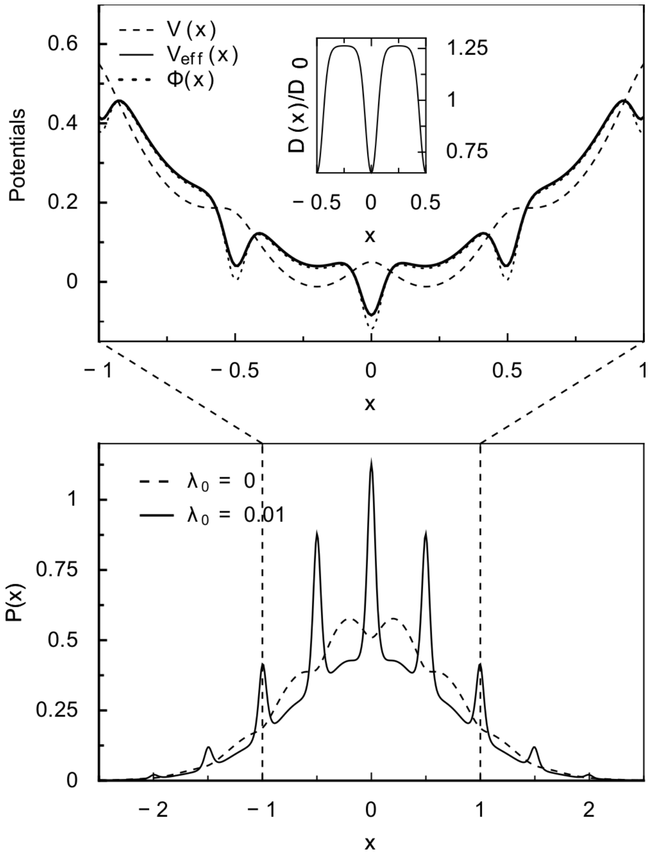

Fig. 1. The quantum Smoluchowski regime compared to its classical counterpart. The potential $V(x)$ and the modified potential $V_{\mathrm{eff}}(x)$ in (2) are shown in the upper panel. In the inset, the modified diffusion function $D(x)$ defined in (4) is depicted. The lower panel shows the stationary probability density $P(x)$ in the classical Smoluchowski ($\lambda_0 = 0$) and quantum Smoluchowski (($\lambda_0 = 0.01$) regimes. Other parameters are set as follows: $x_e = 0$, $T_0 = 0.5$, $\varepsilon = 2$, $\alpha = 2$, $p = 0.5$, $k_0 = 1.0$ and $k_F l = 0.001$

Bistable systems are natural candidates for qubits. Famous examples are Josephson junction based devices which can be generally divided into two classes: charge and flux qubits [3]. It seems that a qubit can also be based on non-superconducting materials [4]. Because within tailored parameter regimes in the quantum Smoluchowski domain there are symmetric peaks in the multistable state, such a system is a good candidate for a *qutrit*. The problem of the qutrit implementation is of a central importance for quantum cryptography [8].

The discussion below is limited to purely qualitative aspects of the qutrit kinetics. We assume for simplicity that there are only three significant (in the statistical sense) peaks in the probability distribution, as e.g. in Fig. 2. Replicating Feynman's discussion of the ammonia molecule [11] one can propose the "Hamiltonian" of the system as a 3×3 real symmetric matrix with diagonal elements proportional to the energy of the system calculated at the magnetic flux extremum value.

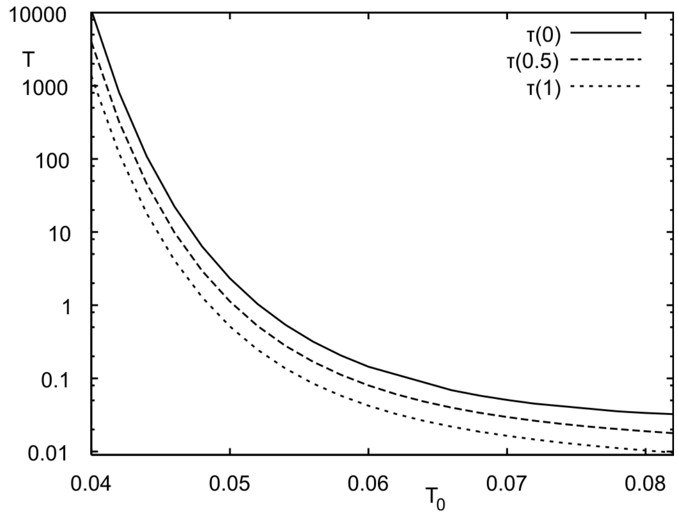

Fig. 2. The lifetime of peaks (log scale) vs. temperature T_0 for the following set of parameters: $\alpha = 0.1$, $\varepsilon = 10$, $k_0 = 0.5$, $\lambda_0 = 0.002$, and $k_F l = 0.001$

An attempt of direct quantization of the classical system which is essentially dissipative (overdamped) suffers serious difficulties. The choice of the quantum Hamiltonian is not obvious. Moreover, this quantum dissipative system is strongly damped. As there is no generic master equation describing reduced dynamics of such a system [12], a phenomenological modelling shall be used. In the regime where only three fundamental peaks are relevant, the presented system is a perfect example of the celebrated *V* system [13], i.e. it consists of three levels: $|x=0\rangle$ and $|x=\pm 1/2\rangle$. The rates of spontaneous transitions can be directly related to the *classical* inter-peak mean passage times $\tau(0 \to \pm 1/2)$ [10]. The transition $|x=\mp 1/2\rangle \to |x=\pm 1/2\rangle$ can clearly be neglected. It is also natural to control the system either by incoherent or coherent driving.

In the following, the simplest case of the dynamics of the incoherently driven system is introduced. We limit our consideration to the dynamic populations i.e. we consider only diagonal elements of the full density matrix of the *V* system. It reads [13]:

$$\frac{d\rho_{11}}{ds} = -\left(S_1 + I_1\right)\rho_{11} + I_1\rho_{00} \tag{8}$$

$$\frac{d\rho_{22}}{ds} = -\left(S_2 + I_2\right)\rho_{22} + I_2\rho_{00} \tag{9}$$

$$\frac{d\rho_{00}}{ds} = -\left(I_1 + I_2\right)\rho_{00} + \left(S_1 + I_1\right)\rho_{11} + \left(S_2 + I_2\right)\rho_{22} \tag{10}$$

with the normalization condition $\sum_i \rho_{ii} = 1$. The parameters I_i and S_i are the rates of induced and spontaneous transitions, respectively. The stationary state [13]:

$$\rho_{11}(s \to \infty) = \frac{I_1(S_2 + I_2)}{S_1(S_2 + 2I_2) + I_2(2S_2 + 3I_2)} \tag{11}$$

$$\rho_{22}(s \to \infty) = \frac{I_2(S_1 + I_1)}{S_1(S_2 + 2I_2) + I_2(2S_2 + 3I_2)} \tag{12}$$

$$\rho_{00}(s \to \infty) = 1 - \sum_{i \geq 1} \rho_{ii}(s \to \infty) \tag{13}$$

Despite its simplicity, the classical treatment described above allows an effective control of populations via a suitable external driving, e.g., finding "equidistribution" conditions $\rho_{ii} = 1/3$, satisfied e.g. for $I_i >> S_i$. The potential application for quantum computing requires also an effective way of controlling coherences. It can be done either phenomenologically via suitable driving inducing inter-peak transitions or by consistent calculations beyond the quantum Smoluchowski expansion resulting in the master equation with non-diagonal elements included. Since the system is mesoscopic we expect the driving to operate at *microwave* [14] rather than optical frequencies.

The approach including kinetics of coherences can be formulated in terms of the phenomenological quantum jump formalism [13]. This problem is postponed to further studies.

Acknowledgement

The work supported by the MNiSzW under the grant No. 202 131 32/3786.

References

[1] HUND F., Ann. Phys. (Leipzig), 32 (1938), 102; KULIK I.O., JETP Lett., 11 (1970), 275; BÜTTIKER M., IMRY Y., LANDAUER R., Phys. Lett., A 96 (1993), 365.
[2] DAJKA J., ŁUCZKA J., SZOPA M., ZIPPER E., Phys. Rev B, 67 (2003), 073305; DAJKA J., KOSTUR M., ŁUCZKA J., SZOPA M., ZIPPER E., Acta Phys. Polon. B, 34 (2003), 3793.
[3] MAKHLIN Y., SCHÖN G., SCHNIRMAN A., Rev. Mod. Phys., 73 (2001), 357.
[4] ZIPPER E., KURPAS M., SZELĄG M., DAJKA J., SZOPA M., Phys. Rev B, 74 (2006), 125462.
[5] ANKERHOLD J., PECHUKAS P., GRABERT H., Phys. Rev. Lett., 87 (2001), 086801.
[6] MACHURA Ł., KOSTUR M., HÄNGGI P., TALKNER P., ŁUCZKA J., Phys. Rev. E, 70 (2004), 031107; ŁUCZKA J., RUDNICKI R., HÄNGGI P., Physica A, 351 (2005), 60.
[7] ANKERHOLD J., Europhys. Lett. 67 (2004), 280; ANKERHOLD J., PECHUKAS P., GRABERT H., Chaos, 15 (2005), 026106.
[8] MOLINA-TERRIZA G., VAZIRI A., URSIN R., ZEILINGER A., Phys. Rev. Lett., 94 (2005), 040501.
[9] DAJKA J., ROGOZIŃSKI S., MACHURA Ł., ŁUCZKA J., Acta Phys. Polon. B, 38 (2007), 1737.
[10] DAJKA J., MACHURA Ł., ROGOZIŃSKI S., ŁUCZKA J., Phys. Rev. B, 76 (2007), 045337.

[11] FEYNMAN R.P., *Lectures on Physics*, Vol. 3, Addison-Wesley, Reading, 1964.
[12] WEISS U., *Quantum Dissipative Systems*, 2nd Ed., World Scientific, Singapore, 1999.
[13] PLENIO M.B., KNIGHT P.L., Rev. Mod. Phys., 70 (1998), 101.
[14] DAJKA J., SZOPA M., VOURDAS A., ZIPPER E., Phys. Rev. B, 69 (2005), 045305; DAJKA J., ZIPPER E., Mater. Sci.-Poland, 24 (2006), 769.

Received 7 May 2007
Revised 19 September 2007